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On Remarkable Properties of Number 2025

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ABSTRACT

In this paper we investigate the number 2025 and visualize its regularity. This is a perfect square, but a deeper look reveals much more structure related to counting lattice points in polygons and polyhedra. We will also discuss the frequency of square and regular years and the uniqueness of such a regular year number.

Key words: figure numbers, trigonal numbers, tetrahedral numbers

MSC2020: 05A17, 52C05

O izuzetnim svojstvima broja 2025

SAŽETAK

U ovom članku proučava se broj 2025 i vizualizira njegova regularnost. Ovaj broj je potpuni kvadrat, a dubljim uvidom otkrivamo bogatu strukturu i povezujemo je s prebrojavanjem cjelobrojnih točaka u poligonalnim domenama. Također dajemo osvrt na frekvenciju pojave kvadratnih i regularnih godina te jedinstvenost ovakve regularne godine.

Ključne riječi: poligonalni broj, trigonalni broj, tetredralni broj

1 Introduction

Number 2025 is very special. It is a perfect square, an odd square, a square of triangular number, sum of two, three and four squares, sum of cubes. It is a regular number, a number that has only 3 and 5 as prime divisors since

$$2025 = 81 \cdot 25 = 9^2 \cdot 5^2 = 3^4 \cdot 5^3$$

and finally, it is a product of squares. In Table 1 we give a (noncomplete) list of sequences that contain 2025 in the Online Encyclopedia of Integer Sequences (OEIS [8]).

Regularity of number 2025 can be visualized by polygonal symmetries connected to number sequences mentioned. Number 2025 belongs to the class of figure numbers, polygonal numbers which are used for counting lattice points inside polygons [1, 10].

In Section 2 we give recursions and graphical representations of some number sequences from Table 1. We show that the predecessor 2024 and successor 2026 are also figure numbers; 2024 is a tetrahedral number (3), while $2026 = 2 \cdot 1013$ is successor of a square number and 1013 is a centered square number, sum of consecutive squares $n^2 + (n+1)^2$ with $n = 22$.

Sequence in OEIS	Occurence	Description of sequence	2025 =
*A*000290	$n = 45$	perfect square	45^2
*A*016754	$n = 22$	odd square (centered octagonal number)	$(2 \cdot 22 + 1)^2$
*A*000537	$n = 9$	sum of cubes (square of triangular number)	$\left(\frac{9 \cdot 10}{2}\right)^2$
*A*001481	$n = 626$	sum of two squares	$27^2 + 36^2$
*A*051037	$n = 109$	5-smooth number (regular number)	$3^4 \cdot 5^2$
*A*238237	$n = 2$	*torn* number	$(20 + 25)^2$
*A*350869	$n = 2$	square of sum of all numbers with one digit	$\left(\frac{(10^1 - 1) \cdot 10^1}{2}\right)^2$

Table 1: *Occurrences of number* 2025 *in OEIS and corresponding formulas*, [8].

n	Prime Factorization	$\varphi(n)$	n	Prime Factorization	$\varphi(n)$
2000	$2^4 \times 5^3$	800	2013	$3 \times 11 \times 61$	1200
2001	$3 \times 23 \times 29$	1232	2014	$2 \times 19 \times 53$	936
2002	$2 \times 7 \times 11 \times 13$	720	2015	$5 \times 13 \times 31$	1440
2003	2003	2002	2016	$2^5 \times 3^2 \times 7$	576
2004	$2^2 \times 3 \times 167$	664	2017	2017	2016
2005	5×401	1600	2018	2×1009	1008
2006	$2 \times 17 \times 59$	928	2019	3×673	1344
2007	$3^2 \times 223$	1332	2020	$2^2 \times 5 \times 101$	800
2008	$2^3 \times 251$	1000	2021	43×47	1932
2009	$7^2 \times 41$	1680	2022	$2 \times 3 \times 337$	672
2010	$2 \times 3 \times 5 \times 67$	528	2023	7×17^2	1632
2011	2011	2010	2024	$2^3 \times 11 \times 23$	880
2012	$2^2 \times 503$	1004	2025	$3^4 \times 5^2$	1080
			2026	2×1013	1012
			2027	2027	2026

Table 2: *Prime factorization and Euler's function* $\varphi(n)$ *for* $2000 \leq n \leq 2027$.

In Section 3 we look at the fact that 2025 is the square of the sum of digits

$$(0+1+2+\cdots+9)^2 = 45^2 = 2025$$

in the decimal representation system, so that it appears in another integer sequence *A*238237 in OEIS [8], sequence of numbers which, when split in two parts of equal length, then added and squared, give the same number,

$$2025 = (20+25)^2. \tag{1}$$

In the last Section 4 we discuss the distribution of integer sequences from the upper part of Table 1 within the natural numbers and show that year 2025 is unique in the following way:

Theorem 1. *Consecutive integers* 2024 *and* 2025 *are the only pair of consecutive integers such that a tetrahedral number is predecessor of an odd square number and they appear with the same index* $n = 22$ *in their respective integer sequences.*

Graphics were made using Geogebra, Rhinoceros 3d with Grasshopper, SAGE [7, 9].

The author would like to thank the referee for many useful suggestions on how to improve the paper.

2 Visualizations of properties of number 2025

2.1 Divisors of 2025

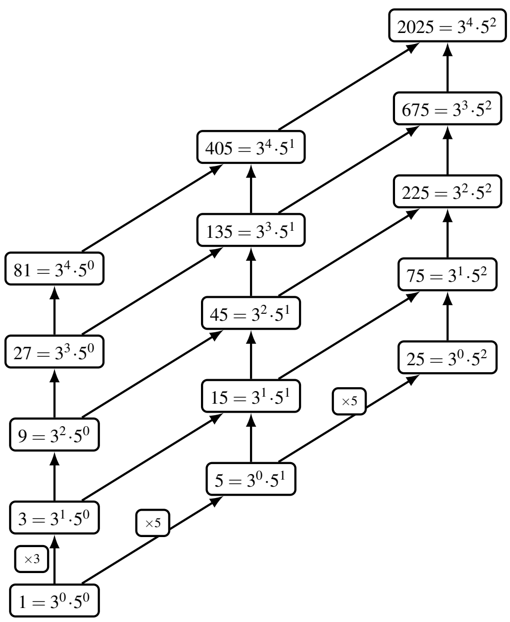

Figure 1: *Hasse diagram of divisors of number 2025.*

Number 2025 has 15 divisors, two of which, 3 and 5, are prime. In Figure 1, we can see the Hasse diagram of divisors of 2025. In Table 2 we see prime factorization and values of Euler function $\varphi(n)$ [3], which represents the number of numbers less than n that are relatively prime to n for $n = 2000, \ldots, 2027$.

Number 2025 is smooth, it has a small set of prime divisors, each of which is itself small. Regular or 5-smooth numbers are numbers with greatest prime divisor less then or equal to 5, so their prime factorization is

$$2^i \cdot 3^j \cdot 5^k, \quad i,j,k \geq 0.$$

As we can see from Table 2, 2025 is the first regular number after 2000 and the next one is $2048 = 2^{11}$, a power of 2. Regular numbers are more frequent within the natural numbers then all other integer sequences from Table 1 which have sparse, polynomial distribution, see (14). Distribution of regular numbers is of order $(\log n)^3$.

2.2 Triangular and tetrahedral numbers

Square root of 2025 is number 45, sum of digits of the decimal system. This number is a triangular number, figure number and first in the class of polygonal numbers counting points in triangular lattices. Triangular numbers also count combinations, T_n is the number of unordered pairs, subsets with two elements, of the set with $n+1$ elements,

$$T_n = \binom{n+1}{2} = \frac{n(n+1)}{2} = 1+2+\cdots+n \qquad (2)$$

and can be visualized by counting integer points in a triangular lattice, Fig. 2.

		$k=2$ Triangular	$k=3$ Tetrahedral		
1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1
1	6	15	20	15	6
1	7	21	35	35	21
1	8	28	56	70	56
1	9	36	84	126	126
1	10	45	120	210	252
1	11	55	165	330	462
		⋮	⋮		
1	24	276	2024	10626	...

Table 3: *Pascal triangle* $P(n,k) = \binom{n}{k}$, $n,k \geq 0$.

They appear in the Pascal triangle, Table 3, as the third diagonal and tetrahedral numbers as the fourth.

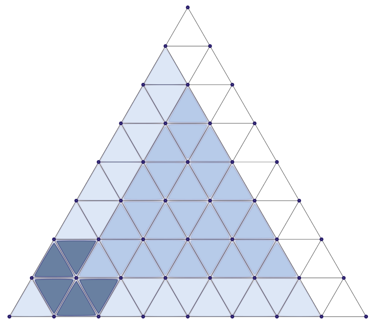

Figure 2: *Triangular lattice with* $T_9 = 45$ *vertices.*

From the triangular lattice in Fig. 2 we can see the recursion of the number of points in the grid, $T_n - T_{n-1} = n$, with the rule of succession is adding a new line with n points as we can see from light blue triangle corresponding to T_8 in Fig. 2. We can also observe hexagons in triangular lattices. The number of hexagons is equal to the number of interior points of the lattice since they are center points of hexagons, so the number of hexagons is the difference of the number of all points and boundary points, $T_n - 3(n-1) = T_{n-3}$.
Sums of first n triangular numbers are tetrahedral numbers ($A000292$ in OEIS), and as the name suggests count points in triangular pyramids, Fig. 3.

$$Te_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \left(\sum_{i=1}^{k} i \right) = \binom{n+2}{3} = \frac{n(n+1)(n+2)}{6}. \qquad (3)$$

Recursion for tetrahedral numbers is $Te_{n+1} = Te_n + T_n$ which can be seen by adding a new horizontal layer to a pyramidal lattice, Fig. 3.

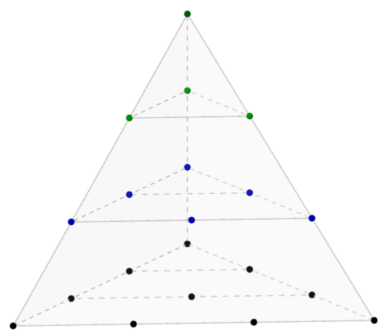

Figure 3: *Tetrahedron with* $Te_4 = 20$ *points.*

2.3 Square numbers

A perfect square is a number of form n^2, $n \in \mathbb{N}$. For square numbers we have the recursive formula

$$(n+1)^2 = n^2 + 2n + 1, \qquad (4)$$

which can be seen visually when counting points in $n \times n$ square lattice. We can divide $n \times n$ points of the lattice into $(n-1) \times (n-1)$ square and $2n+1$ points that will make the last row and column, Fig. 4. We see that square numbers are polygonal numbers, used to count points within a polygonal area. We can also interpret them through metrics, so that n^2 is area of a square with side length n.
From the recursive formula (4) we see that we can represent n^2 as sum of the first n odd numbers

$$n^2 = \sum_{k=1}^{n} (2n-1). \tag{5}$$

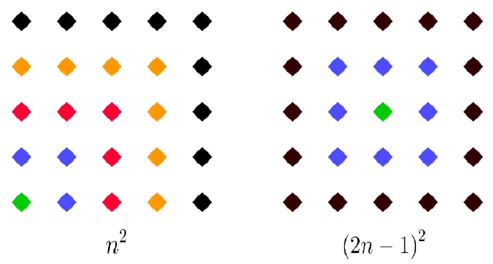

Figure 4: *Recursive generation of square and odd square numbers as Pythagorean gnomons.*

On the other hand, we can divide integer points in a square area into two triangular areas, and the number of integer points satisfy $T_n + T_{n-1} = n^2$. In Fig. 5 we see $2025 = T_{45} + T_{44} = 1035 + 990$ points divided in two triangular parts of the square with side length 45.

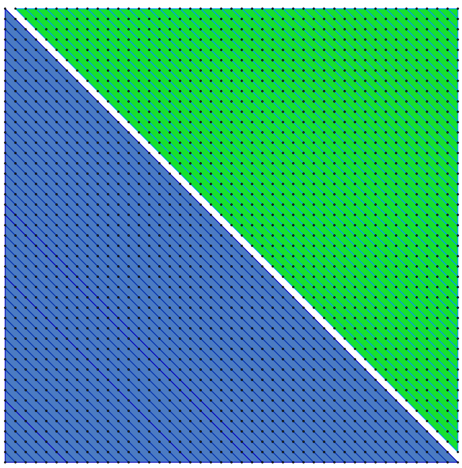

Figure 5: *2025 points in a square grid divided into two triangular areas.*

There is another interesting appearance of triangular numbers. If we look at $n \times n$ square grid, then T_n is number of rectangles found in it. A rectangle is defined by two horizontal and two vertical lines, so number of rectangles is equal to square of number of pairs chosen from set of $n+1$ lines found in the square grid by the product rule.

2.4 Square of triangular number is sum of cubes

Number properties, especially of harmonic whole numbers or 3-smooth numbers were studied from ancient times and the following formula for the sum of first n cubes was a result from the Pythagorean school. Nichomachus' Theorem states

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2 \tag{6}$$

and today this is special case of Faulhaber's formula, [6] saying that sum of p-th powers of first n numbers is a rational polynomial in n of degree $p+1$. Coefficients of this polynomial were first found by Jakob Bernoulli while studing these *Sumas potestaum* and they are now called Bernoulli numbers B_n,

$$\sum_{k=1}^{n} k^p = \frac{1}{p+1} \sum_{i=0}^{n} \binom{p+1}{i} B_i n^{p+1-i} \tag{7}$$

For $p = 1$ the sums are triangular numbers, for sum of squares $p = 2$ we have square pyramidal numbers and for $p = 3$, (6) squares of triangular numbers.
One interesting formula for Bernoulli numbers is as follows, [2]

$$B_n = \frac{2n!}{(2\pi)^n} \left(\frac{1}{1^n} + \frac{1}{2^n} + \frac{1}{3^n} + \ldots \right). \tag{8}$$

When we want to visualize eq. (6), we can look at cubes as volumes of solid bodies or as number of points inside cubes. We can do the same for planar formulas we have seen earlier, so n^2 is area of a square of side with length n, or number of integer points in a $(n-1) \times (n-1)$ grid, since division of a side of length n will yield $n+1$ point. We can see in Fig. 5 how union of two triangles in a square will not give the same area. Relationship of the area A of polygonal domain with integer coordinates of vertices and the number of integer points in the interior I and number of points that lie on the boundary B is given by Pick's theorem [1]

$$A = I + \frac{B}{2} - 1. \tag{9}$$

For counting points in polyhedrons there is no simple formula that connects discrete and continuous volume. This is a wast subject of enumerative geometry known as Erhart theory,[1]. In our case we can see this issue. For the cube, volume a cube with one vertex at origin point of side length n will give n^3 and number of integer points is $(n+1)^3$. But if we instead look at the trirectangular tetrahedron inside a cube, that contains one cube corner, then volume is $n^3/6$. Using eq. (6), we have that volume of union of 9 cubes with side lengths from 1 to 9 is 2025, Fig. 6, and the volume od their tetrahedrons is $2025/6 \approx 337.5$. If we look at integer points in the cubes, there will be 3025 points and in tetrahedrons we have Te_n points so we have total of

$$1+4+10+20+35+56+84+120+165 = 495$$

points. Ratios of discrete and continuous volumes is $(1+\frac{1}{n})^3$ for cubes and $(1+\frac{6}{n}+\frac{11}{n^2}+\frac{6}{n^3})^3$ for tetrahedrons, so we see that in tetrahedral lattices integer points are denser then in the cubic ones.

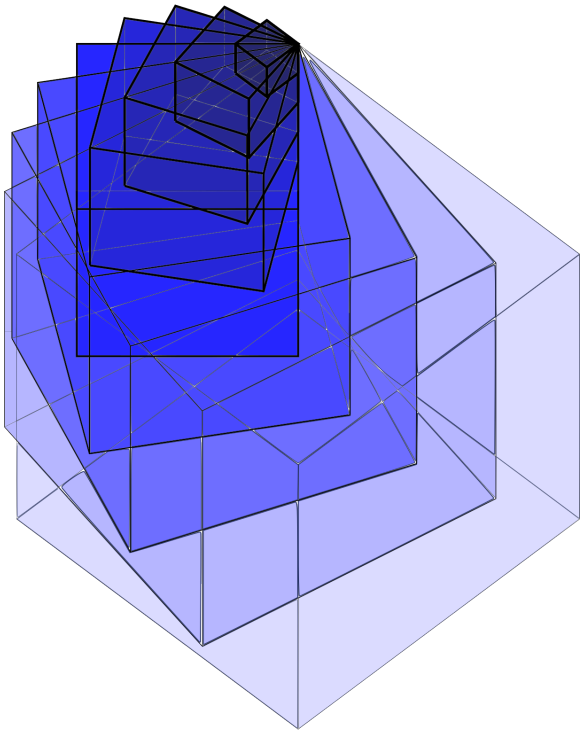

Figure 6: *Nine cubes with sum of volumes equal to 2025.*

2.5 Odd squares

Odd square numbers can be written in the form $(2n+1)^2$ for $n \geq 0$. Then 2025 is an odd square for $n = 22$. These numbers are also called centered octagonal numbers because they count points lying on sides of octagons together with the central point, Fig. 7.

$$(2n+1)^2 = 4n^2+4n+1 = 4n(n+1)+1 = 8\frac{n(n+1)}{2}+1 \quad (10)$$

Here we have the recursion $(2n+1)^2 = (2n-1)^2+8n$ so adding $8n$ new points gives the next odd square, Fig. 4. For number 2025 we have:

$$2025 = 8\cdot 253+1 = 8(22\cdot 23)+1 = \sum_{k=1}^{22} 8k+1. \quad (11)$$

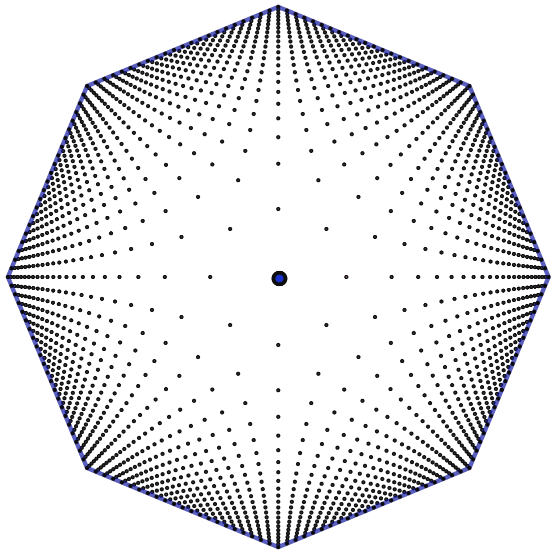

Figure 7: *2025 points in centered octagonal arrangement.*

In Fig. 7 we can see the 2025 points from eq. (11). The points are lying on 22 octagons, where k-th octagon has a side divided into k parts, which together with the central point gives 2025 points. Counting using symmetry, it suffices to count the points strictly inside an angle of $\left[0, \frac{2\pi}{8} = \frac{\pi}{4}\right\rangle$, which would be $1+2+\cdots+22$.
When we remove the central point, the number of remaining points gives a tetrahedral number $2024 = 8\cdot 11\cdot 23$.
Here the symmetry group is the dihedral group D_4 of order 8, since the area inside the angle of $\left[0, \frac{\pi}{4}\right\rangle$ can be rotated four times for the angle $\frac{\pi}{4}$ and then mirrored by the horizontal axes, which is the same symmetry of the Hasse diagram in Fig. 1, having 8 parallelograms or the symmetry of coordinate system in space, where one octant generates the space with four rotations and one reflection.

3 Interesting number digits properties of 2025

Number 45 is the sum of all numbers written with one digit,

$$45 = 0+1+2+3+4+5+6+7+8+9.$$

In general, the sum of all numbers with the number of digits less or equal to n ($A037182$ in OEIS) is

$$a(n) = \frac{10^n\cdot(10^n-1)}{2}. \quad (12)$$

The sum of all number with at most two digits is the sum of all number less then 100 which is $4095 = 99\cdot 100/2$.
Then 2025 is the smallest square of such a number ($A350869$ in OEIS), which has interesting consequences in linking operations on numbers (addition, multiplication) and concatenation of digits to represent numbers.

Namely, 2025 is a *torn* number, ($A238237$ in OEIS), when we split it in two equal parts, add them and then square we arrive at 2025 again,

$$2025 = 45^2 = (20+25)^2. \tag{13}$$

Another example of a torn number is $3025 = 55^2$, the next square of the triangular number $T_{10} = 55$. These numbers are also called Kaprekar numbers [5]. We have $2025 = 20 \cdot 10 + 2 \cdot 10 + 5 = 2 \cdot 10^2 + 2 \cdot 10 + 5$.
There is another digit property, increase digit square preserving property. Namely $2025 = 45^2$ and increasing each digit by 1 gives $3036 = 56^2$. If we denote the decimal representation of a number $\overline{abcd} = a \cdot 10^3 + b \cdot 10^2 + c \cdot 10 + d$, then $\overline{ab} - \overline{cd} = 5$ for 2025 and 3136. We have

$$n^2 = 100ab + cd = 100ab + ab + 5 = 101ab + 5$$

so the prime number $101 = 45 + 56$ gives remainder 5 when dividing n^2.

4 *Square* and other special years

Since we are now in the year 2025, we can ask how frequent square years are, and the answer is given by the distribution of square numbers in natural numbers. Since the difference of two consecutive squares is $2n+1$, it is clear that the gaps increase. Last square year was $44^2 = 1936$, so $2025 = 1936 + 2 \cdot 44 + 1$ comes 89 years later and for the next one we will wait 91 years. Square years are currently in a *once in a lifetime* frequency and it is decaying. In Table 2 we have prime factorization for year numbers since the last regular year 2000.
We list some sequence elements neighboring 2025 for integer sequences from Table 1:

- perfect squares

  $$\ldots 1849, 1936, n = 45 : 2025, 2116, 2290 \ldots$$

- odd squares (centered octagonal numbers)

  $$\ldots 1681, 1849, n = 22 : 2025, 2209, 2401 \ldots$$

- squares of triangular numbers

  $$\ldots 784, 1296, n = 9 : 2025, 3025, 4356 \ldots$$

- regular numbers

  $$\ldots 1944, 2000, n = 109 : 2025, 2048, 2160 \ldots$$

- *tetrahedral numbers*

  $$\ldots 1330, 1540, 1771, n = 22 : \mathit{2024}, 2300, 2600, 2925 \ldots$$

- *centered square numbers* $4n^2 + 4n + 2$

  $$\ldots 1522, 1682, 1850, n = 22 : \mathit{2026}, 2210, 2402, 2602 \ldots$$

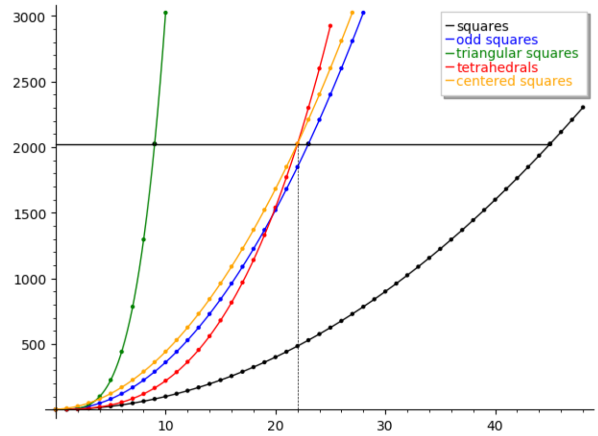

Figure 8: *Graphs of* x^2, $(2x+1)^2$, $\left(\frac{x(x+1)}{2}\right)^2$, $\frac{x(x+1)(x+2)}{6}$ *and* $(2x+1)^2 + 1$.

The odd squares are subset of squares so they are twice less frequent. The fastest growing sequence are squares of triangular numbers, their distribution is of order n^4.
Distribution of regular numbers within natural numbers differs from other sequences in Table 1; square numbers have polynomial growth while regular numbers are more dense and follow logarithmic growth. Some bounds regarding distribution of regular numbers can be found in [4], here is one explicit bound

$$\frac{1}{k!} \prod_{p \leq y} \frac{\log x}{\log p} \geq \Psi(x,y) \geq \frac{1}{k!} \prod_{p \leq y} \frac{\log X}{\log p} \tag{14}$$

for $x \geq y \geq 2$ and $x \geq 4$, where $\Psi(x,y)$ is number of integers less of equal to x that are y-smooth and $X = \prod_{p \leq y} p$.
Next regular year will be 2048 but all other square or regular years will happen in the next century.
Another interesting question can be posed about consecutive regularity we see in $2024- > 2025- > 2026$, tetrahedral number as predecessor of square of triangular number and its successor 2026 is two times centered square number $C_{4,n} = (2n+1)^2 + 1$ for $n \geq 1$, ($A069894$ in OEIS). It is also worth mentioning the Pythagorean triple $(45, 1012, 1013)$ which is representation of 2025 as difference of squares, $2025 = 1013^2 - 1012^2$ and another one $(27, 36, 45)$ which represents 2025 as sum of squares. The perimeter of that triangle with legs 27 and 36 and hypotenuse 45 equals 108 and division into triangle sides will yield ratio of $3:4:5$, so this is a derived triple $9\dot{(}3,4,5)$. This year we even had 'Pythagorean date' 16.9.'25 that relates to the first triple $(3,4,5)$.

Now we will prove that $(2024, 2025)$ is the only pair of consecutive numbers such that a tetrahedral number followed by an odd square number at the same index in respective sequences. The reason is that 2024 is the last tetrahedral number that is less then an odd square number in the same index.

Proof of Theorem 1 From Fig. 8 we can see that around $n = 22$, which is index of 2025 as odd square, 2024 as tetrahedral and 2026 as twice the centered square number is exactly where twice the centered square number become greater then tetrahedral number of the same index. Intersection of graphs for $\frac{x(x+1)(x+2)}{6}$ and $(2x+1)^2+1$ in Fig. 8 is

$$\alpha + \frac{169}{3\alpha} + 7 \approx 22.023$$

for $\alpha = \left(\frac{1}{9}\sqrt{219129}+426\right)^{\frac{1}{3}}$. For $n = 22$ we have $Te_{22} = 2024$ and $(2\cdot 22+1)^2 = 2025$, so

$$\frac{n(n+1)(n+2)}{6} = (2n+1)^2 - 1$$

has an integer solution. All greater tetrahedral numbers will be greater then odd squares, as we can see in Table 4, the difference of odd squares and tetrahedral numbers is $\frac{k(k+1)(22-k)}{6}$ and is zero for $k = 22$. □

From Table 4 we see two other consecutive pairs of tetrahedral numbers 9 and 2600, with odd square successors 10 and 2601 respectively, where the indices differ. There is another pair of consecutive numbers, tetrahedral $Te_5 = 35$ and even square $(T_3)^2 = 36$ which also have different indices in corresponding sequences.

We end with final remark that year 2025 is numerically a unique moment in history!

k	$(2k+1)^2$	Te_k	$-\frac{k(k+1)(k-22)}{6}$
0	1	0	1
1	**9**	1	8
2	25	4	21
3	49	**10**	39
4	81	20	61
⋮	⋮	⋮	⋮
21	1849	1771	78
22	**2025**	**2024**	1
23	2209	2300	-91
24	2401	**2600**	-199
25	**2601**	2925	-324
26	2809	3276	-467

Table 4: *Odd squares, tetrahedral numbers and their difference.*

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