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## Road and Rail Infrastructure III

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# OPTIMISATION OF RAILWAY OPERATION BY APPLICATION OF KRONECKER ALGEBRA 

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#### Abstract

Kronecker Algebra consists of Kronecker Product and Kronecker Sum. This mathematical model can be used to model systems consisting of a number of limited resources and several actors. In particular, it can be used to model railway systems with trains, track sections and their routes. In this paper we show several applications of Kronecker Algebra in the railway domain. We consider deadlock prevention, travel time calculation, and energy analysis. The integration of these three tasks within one single type of Kronecker-based analysis is rather simple and can be carried out very efficiently. Due to the fact that Kronecker Algebra operations can be easily parallelized, our implementation can take full advantages of today's multi-core computer architecture. In addition our implementation shows that adding constrains for connections or overtaking speeds up the calculation. In fact, a harder problem is easier to solve.


Keywords: Kronecker Algebra, railway systems, deadlock, travel time, energy analysis

## 1 Introduction

We present a graph-based method for analysis and optimization of railway networks on a fine-grained level. The routes of trains within this network are modelled by graphs. Each route consists of at least one track section and each track section may be part of several routes. We assume that at the same time only one single train occupies a track section. Our model employs semaphores [1] in the sense of computer science to guarantee that only one train enters a track section. The semaphores are also modelled by graphs. Each graph can be represented by its adjacency matrix. Simple matrix operations can be used to model concurrency and synchronization via semaphores. These matrix operations are known as Kronecker Sum and Kronecker Product and are part of the so called Kronecker Algebra. By applying these operations, we can get a matrix describing the whole railway system; in particular it contains all movements of the trains within this network. The whole theoretical background, including several examples of the applications of Kronecker Algebra, was published in previous papers [2], [3], and [4].This paper contains some advanced examples.

## 2 Example

Figure 1 shows a railway network with 9 track sections. Our example contains four trains which have the following routes:

- Train 1 (T1): 1-5-7 (p5, v1, p7, v5, v7)
- Train 2 (T2): 2-4-5-6-9 (p4, v2, p5, v4, p6, v5, p9, v6, v9)
- Train 3 (T3): 3-4-5-6-8 (p4, v3, p5, v4, p6, v5, p8, v6, v8)
- Train 4 (T4): 8-6-5-1 (p6, v8, p5, v6, p1, v5, v1)


Figure 1 Railway system


Figure 2 Resulting graph (start node is the most left node)

The resulting graph of this example is illustrated in Figure 2. It contains 802 nodes, including yellow, red and green nodes, which have the following meaning:

- Red nodes denote deadlocks. These nodes are eliminated before the following calculations because we are interested in avoiding deadlocks.
- Green nodes denote safe states. A state is safe if all trains can perform their actions without having to take into account the moves of the other trains in the system, provided that the track section which they are to enter is not occupied by another train. If a track section is occupied by another train, the movement of the train wanting to enter may be delayed (blocked) but no deadlock can occur.
- From yellow nodes both red and green nodes can be reached.

Due to the size of the resulting graph, the node-ids and the edge labels are removed for a clear illustration. In general, each edge is labelled by the actions of the trains (cf. [2], [3], and [4]). Our implementation of Kronecker Algebra is very efficient in space and time, because we use a lazy implementation, where only the reachable parts of the matrix is calculated (cf. [3]). The calculation of the resulting graph of our example takes about $30 \mu \mathrm{~s}$. As we are interested in the calculation of the worst-case travel time of each train within the railway network we define the following travel times for the trains on the track sections (Table 1). The columns labelled with " 1 " to " 9 " denote the track section.

Table 1 Travel time values

| Train | $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T1 | 5 |  |  |  | 5 |  | 5 |  |  |
| T2 |  | 2 |  | 3 | 5 | 3 |  |  | 2 |
| T3 |  |  | 2 | 3 | 5 | 3 |  | 2 |  |
| T4 | 3 |  |  |  | 3 | 2 |  | 1 |  |



Figure 3 Reduced graph
Assuming that each train is independent from the others, the complete travel time is the sum of the travel times of all track section of the train's route. As there exist several common used track sections, there is an influence of the travel time of the trains, which use the track
sections. Further information about the synchronization between trains and the effect on their travel time can be found in [3] and [4]. The resulting travel time of the trains can be found in the second and third column of Table 2. The graph can be reduced to its relevant synchronizing nodes. Due to space restriction, the reduction algorithm is not explained here in detail. The reduced graph of our example, including the travel time values for the four trains at the nodes and along the path between them can be found in Figure 3.
In our example we used track sections as commonly used resources. Our model can be extended to use other shared resources, for example available energy (cf. [5], [6], and [7]). The available energy is quantised into standardised packages, e.g. 1 MWh . We can model a power station or substation capable of producing e.g. 20 MW by using a counting semaphore of size 20 (cf. [5], [6], and [7]).
Now we extend our model. For each train it is known a priori, how much energy is needed on a track section. This amount of energy is reserved before entering the track section and released afterwards. Modelling available energy and required energy in this way ensures that a train needing more energy than can be delivered by the power station is blocked. In particular, it will stop its journey and can continue when enough energy is available (e.g. because another train has released its energy needs or the station can provide a higher number of energy packages). Table 2 shows the energy demand of the four trains within our railway system and their needs of energy for each track section.

Table 2 Energy demand

| Train | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T1 | 2 |  |  |  | 2 |  | 2 |  |  |
| T2 |  | 1 | 1 | 2 | 1 |  | 1 |  |  |
| T3 |  |  | 1 | 2 | 1 | 1 |  |  |  |
| T4 | 2 |  |  |  | 2 | 1 |  | 1 |  |

It is obvious, that the maximum amount of energy at a given time is eight. If the station can deliver at least eight energy units, there is no effect on the travel time of the four trains (Table 3 , fourth column). The size of the graph increases to 6443 nodes, because an additional counting semaphore of size 8 is added to our calculations. The influence on the travel time can be found in columns five, six, and seven of Table 3, if there are only six, four, or two energy units available, respectively. Obviously, the travel time increases because some of the trains will have to stop and wait until enough energy is available to continue their travel. The size of the resulting graph decreases, because we specify the problem in more detail and thus the number of possible train movements decreases which has an effect on the size of the graph. By application of Kronecker Algebra, a harder problem is easier to solve. The last column of Table 3 shows the result, if only one energy unit is available. As a consequence, no train can finish its journey, because each of them will need at least two available energy units for particular track sections. The resulting graphs for two and one available energy units are given in Figures 4 and 5.

Table 3 Resulting travel time

| Train | Travel Time | Worst cast <br> travel time | Travel time <br> $\mathrm{E}=8$ | Travel time <br> $\mathrm{E}=6$ | Travel time <br> $\mathrm{E}=4$ | Travel time <br> $\mathrm{E}=2$ | Travel time <br> $\mathrm{E}=1$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T1 | 15 | 25 | 25 | 30 | 30 | 30 | $\infty$ |
| T2 | 13 | 36 | 36 | 41 | 49 | 52 | $\infty$ |
| T3 | 15 | 38 | 38 | 43 | 51 | 54 | $\infty$ |
| T4 | 9 | 28 | 28 | 33 | 39 | 39 | $\infty$ |
| No. of nodes | 802 | 802 | 6443 | 6053 | 3147 | 418 | 11 |



Figure 4 Resulting graph (two energy units)


Figure 5 Resulting graph (one energy unit)
In Figure 5, it can be seen that each train can perform its first action, but then it deadlocks, due to the lack of available energy. In our examples we have used integer values for travel time values. Extensions of our model, e.g. with decimal values or taking braking and acceleration time into account in case of blocking are possible, but not discussed here.

## 3 Conclusion

We have presented a practical example for the application of Kronecker Algebra, where we model the movements of trains within a railway system and the access on a shared track section. Afterwards the travel time of each train can be calculated. We extended the example by the modelling of energy units, produced by a power station and analyzed the results, based on the available amount of energy. Our approach can be used to model complex railway systems including aspects of being deadlock-free, being conflict-free, and being minimal in terms of energy demand. The theoretical background of Kronecker Algebra can be found in some preliminary papers ( [2], [3], [4]. [7])

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