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Road and Rail Infrastructure II
Stjepan Lakušić – EDITOR

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Abstract

The Czech railway infrastructure manager has described design principles and recommendations for installation of continuous welded rail on bridges in a regulation which summarizes experience of bridge – rail interaction since the continuous welded rail was introduced as a common structure. The regulation covers the most common cases of bridge – rail interaction concerning the type of a bridge structure, deck and bearing arrangement as well as the design of rail superstructure components.

The bridge in Usti nad Orlici is situated on the main railway line Berlin – Prague – Brno – Vienna/Bratislava. Construction of the new bridge is a part of the railway station reconstruction project focused on speed increase. The design of bridge supports position had to take into account the situation of the confluence of two rivers and a road which led to designing a of nonstandard arrangement of the bridge bearings. This design required extra attention to an assessment of the particular bridge – rail interaction.

Two approaches to evaluation of the bridge – rail interaction were applied. The analytical solution consisted in solution of the system of differential equations describing the thermic interaction. Although the experience with the analytical solution is very good it doesn’t included curvature of rail and the bridge as it was in this case. Therefore the finite element analysis was also performed in purpose to compare either results or estimate corrective factors or a modification of input parameters for the analytical method.

Keywords: continuous welded rail, bridge rail interaction, bridge thermic expansion, track analysis

1 Introduction

The thermic interaction bridge structure and continuous welded rail is a subject of investigation these days. Infrastructure managers would like to install continuous welded rail whenever track parameters permit it in the aim to significantly reduce the maintenance demands and costs. The only limiting parameter defined in the Czech regulation for the track outside bridges is value of curve radius 200 m when tracks are reconstructed or modernized. Installation of continuous welded rail according to the new draft of the Czech Infrastructure Administration regulation would be acceptable even in smaller radii for particular conditions. Needs to evaluate a number of cases of application of continuous welded rail on bridges in track sections, in which continuous welded rail was not possible to install previously, have arisen. A clear and unambiguous method how to evaluate the interaction between bridge and rail is necessary for designers and infrastructure administration. It was found out that calculation
according to EN 1991-2 Eurocode 1 'Actions on structures', Part 2 'Traffic load on bridges' doesn't comply with the regulation of Railway Infrastructure Administration in the Czech Republic. It must be pointed out that the provisions of continuous welded rail installation on bridges are based on experience in long term behaviour of the interaction which is permitted to take into account according to the Eurocode.

The theoretical base of thermic interaction between bridge and rail will be described on the necessary level. The application of the theoretical base into calculations of permissible expansion length of bridges regarding effects in rail will be explained. The bridge in Usti nad Orlici which is situated on the main railway line Berlin – Prague – Brno – Vienna/Bratislava is an example of design of non-standard assembly of bridge expansion length and bearings. The differences in the interaction evaluation according these two different approaches are commented in the paper.

2 Theoretical description of the interaction between bridge and continuous welded rail

The application of continuous welded rail is based on the presumption that axial forces in a section of continuous welded rail, in which the whole track or rails don’t move (more precisely \( \frac{du}{dx} = 0 \)), are proportional to thermic load and are independent on track length, which can be expressed, see for details [1]:

\[
N_x = E \cdot A \cdot \left( \frac{du}{dx} - \alpha \cdot \Delta T \right)
\]

in which \( N_x \) [kN] is axial force in continuous welded rail, \( E \) [Pa] is Young’s modulus, \( A \) [m²] is area of rail cross section of both rails, \( u \) [m] is longitudinal displacement of track (rails or track length), \( \alpha \) [K⁻¹] coefficient of thermic expansion of rail, \( \Delta T \) [K] temperature difference between actual and neutral temperature of rails.

Additional longitudinal load originates from accelerating or breaking forces of rolling stock. It was confirmed by calculations, practical experience and measuring that track is able to transfer and resist this load and as a structure it is safe and reliable.

The thermic expansion of bridge structure is another additional load causing on continuous welded rail. A basic question is how big expansion length is acceptable from point of view of the ability of track on the bridge to resist all longitudinal forces. When describing continuous welded rail on bridge we should assume the longitudinal displacement of track caused by the thermic expansion of bridge structure. An analogous situation of the track on subgrade in a cut or on an embankment is the breathing length of continuous welded rail where longitudinal displacements occur. They are not homogeneous (\( \frac{du}{dx} \neq 0 \)) and forces towards the rail end (rail joint or expansion joint) decrease. Mathematical description expresses the fact that changes of longitudinal forces along the track length is proportional to longitudinal resistance activated in a particular cross section and a load by acceleration or breaking of vehicles, which is expressed by a simple formula:

\[
\frac{dN_x}{dx} = r_x - q_x
\]

in which \( r_x \) [kN.m⁻¹] is longitudinal resistance per meter, \( q_x \) [kN.m⁻¹] is longitudinal load per meter caused by acceleration or breaking.
The key parameter for the description of behaviour of continuous welded rail is longitudinal resistance of track either against subgrade or bridge structure. Longitudinal resistance is generally nonlinear function of longitudinal displacement of rail $u$, see Fig. 1. Calculation of longitudinal displacement $u$ of continuous welded rail along the track length on subgrade is substantial for determination of axial forces $N_x$ in continuous welded rail. The longitudinal resistance of continuous welded rail on subgrade is often simplified as a constant value $r_0$ which is independent on longitudinal displacement value. This approach can’t be applied to track on bridge because the fact that forces transferred from bridge to rail would be constant and wouldn’t be dependent on thermic expansion of bridge structure. That is why the conservative approach (on safe side) is usually introduced in which longitudinal resistance $r_x$ is modelled by a linear function of longitudinal displacement $u$, for track on the bridge of relative displacement bridge – track:

$$r_x = k \cdot u$$  \hspace{1cm} (3) \\
$$r_x = k_b \cdot (u - u_b)$$  \hspace{1cm} (4)

in which $k$ [kN.m$^{-2}$] is constant expressing linear dependence on longitudinal displacement $u$, $k_b$ is constant expressing linear dependence on longitudinal relative displacement $(u - u_b)$ [kN.m$^{-2}$].

Thermic expansion of bridge structure is described by $u_b$ which can be calculated when omitting influence of rail on bridge because of much smaller cross section area of rails:

$$u_b = x_b \cdot \alpha_b \cdot \Delta T_b$$  \hspace{1cm} (5)

in which $u_b$ [m] is longitudinal displacement caused by thermic expansion, $\alpha_b$ [K$^{-1}$] is coefficient of thermic expansion of bridge structure, $\Delta T_b$ [K] is temperature difference between actual temperature of bridge and bridge temperature at continuous welded rail installation, $x_b$ [m] is distance from longitudinal rigid bridge bearings.

The complete solution was worked out and published by prof. Fryba in [2]. Behaviour of continuous welded rail on bridge can be described by differential equations evaluated as a combination of Eqs. (1) – (4):

$$-EA \frac{d^2u}{dx^2} + k \cdot u = q_x$$  \hspace{1cm} (6) \\
$$-EA \frac{d^2u}{dx^2} + k \cdot (u - u_b) = q_x$$  \hspace{1cm} (7)

Differential Eqs. (6) and (7) are written for track sections in bridge vicinity and for every expansion structure of the bridge. Solution of these equations is both the longitudinal displacement...
of rail on subgrade and on bridge and consequently the axial forces in track, solving of the equations is described in [2] or [3]. Function continuity is ensured by calculation of integration constants in the solution.

Input parameters are essential for the evaluation of the bridge track interaction. Prof. Fryba in [2] provides a comprehensive system of the input parameters, e.g. values of longitudinal track resistance in common conditions and separately in winter season, equivalent coefficient of thermic expansion \( a_b \) for different kinds of bridge structures, calculation of critical forces and stresses in rails.

Equivalent coefficient of thermic expansion \( a_b \) expresses the experience with bridge expansion which is usually less than calculated for \( a_b \) related to a structural material and maximum and minimum temperatures. For example the coefficient \( a_b \) equals 6 kN.m\(^{-1}\) according to [2] for steel bridge structure with ballast bed, which is the bridge in Usti nad Orlici.

![Figure 2](image)

**Figure 2** Thermic expansion of the bridge structure of Znojmo viaduct (slope coefficient is reciprocal to \( a_b \))

The coefficient \( a_b \) was evaluated from recently finished continual monitoring of Znojmo viaduct (but there is not continuous welded rail), which is also a steel bridge with ballast bed. The coefficient \( a_b \) was calculated by regression analysis for values of longitudinal expansion and temperature of the bridge structure stored every second during the two years monitoring. The review of analysis results is in the table 1 from which it is evident that the value of \( a_b \) is bigger than 6 kN.m\(^{-1}\). The regression of values stored in April of 2011 is shown on Fig. 2. The value of coefficient marginally varies in particular months, see Fig 2.

**Table 1** Results of regression analysis of thermic expansion of Znojmo viaduct

<table>
<thead>
<tr>
<th>Length of the bridge [m]</th>
<th>Coefficient ( a_b ) [10-6K(^{-1})]</th>
<th>Temperature range [K]</th>
<th>Calculated expansion [mm]</th>
<th>Measured expansion [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>220,97</td>
<td>9,7</td>
<td>51,4</td>
<td>110</td>
<td>111</td>
</tr>
</tbody>
</table>
3 Example of analysis of bridge – rail interaction

The solution of the bridge in Usti nad Orlici which is situated on the main railway line Berlin – Prague – Brno – Vienna/Bratislava is an example of analysis. The design of the position of bridge supports had to take into account the situation of the confluence of two rivers and a road which led to a design of nonstandard arrangement of the bridge bearings. The bridge – rail interaction was analysed both by analytical method and finite element method because of track in curve. The double track on the bridge is situated in the curve of radius 755 m. The ballast bed is designed continuously over the bridge. The viaduct is composed of three bridge structures separately expanded due to thermic loads. The main reason for the individual evaluation of the bridge structures and tracks on them was an untypical assembly of longitudinally movable bridge bearings of two structures opposite each other on the support and also the expansion length of structures over the limits according to the infrastructure administration regulation.

3.1 Analytical solution

The analytical approach simplified the bridge structure as a system of interacting straight beams. The longitudinal resistance was substituted by a system of linear springs. The analytical calculations were based on the solving of the Eqs. (5), (6) and (7). The results of the solution were functions $u$ and $u_b$ which are used for calculation of axial forces in track $N_x$ according to Eq. (1) and subsequently of stresses in rails $\sigma_x$. The values of displacements, forces and stresses were evaluated according to the Czech regulation by checking of following criteria:

- the forces in track caused by the maximum temperature must be lower than a half value of the critical force of track buckling;
- the additional stresses in rails originating from the bridge track interaction are included to the evaluation of rail stresses, rail wear is important;
- gap between ends of rail after accidental rail break must be less than 50 mm due to the minimum temperature load during the winter season; a vehicle could safely pass over a gap of this size;
- forces may not cause a damage of track components, especially rail fastening;

The analysis results of displacements of the bridge and rail in Usti nad Orlici are shown on Fig. 3, the results of axial forces are shown on Fig. 4. The comparison of results obtained by the analytical analysis and the analysis by Finite Element Method (FEM) are presented on both figures.

![Figure 3](image-url)  
Figure 3  Longitudinal displacements for the maximum temperature
3.2 Finite element method analysis

The analytical methods of investigations of thermic interaction of bridge structure and continuous welded rail do not allow by a simple method taking into account a track in horizontal curve. That is why the FEM analysis, which permits to include this structure geometry, was done in parallel [4].

The horizontal geometry of the bridge structures was modelled, see Fig. 5. The longitudinal resistance was expressed by nonlinear contacts between ballast bed and the steel structure. On the other hand simplified boundary conditions at rigid bridge bearings and at the boundaries of the track model were included in the FEM model. Also model contended simplifications of subgrade and the bridge supports elasticity in longitudinal direction.

When comparing results of the presented analytical method and FEM it can be state a good correspondence between both results. An influence of horizontal curve is marginal concerning lateral parameters of the bridge structures. Disturbances in FEM analysis are evident. These are caused by a particular interaction of track and ballast bed with components of the steel structure.
Conclusion

Every bridge structure continuous welded rail is constructed on should be evaluated according to specifications of EN 1991-2 Eurocode 1 'Actions on structures', Part 2 'Traffic load on bridges'. This standard provides basic values of input parameters necessary for calculations of bridge rail interaction and analyses guidance. If the Eurocode parameters were used the evaluation of bridges would not be in compliance with experience of the Czech Railway Infrastructure Administration expressed in the regulations then. Besides the evaluation criteria are not the same in both standards which is confusing both for designers and administration. The only possibility how to solve these discrepancies between specifications in standards is use of calculation input parameters verified by years’ experience and confirmed by monitoring. Currently a few projects focused on monitoring of temperature loads, expansion of bridge structures and consequently behaviour of continuous welded rail are in process in the Czech Republic.

On the other hand it is evident from the comparison of interaction calculations that an analysis method doesn’t play an essential role neither in case of the bridge in a curve nor in modelling of longitudinal resistance as linear or nonlinear.

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References


