Distribuirane sile

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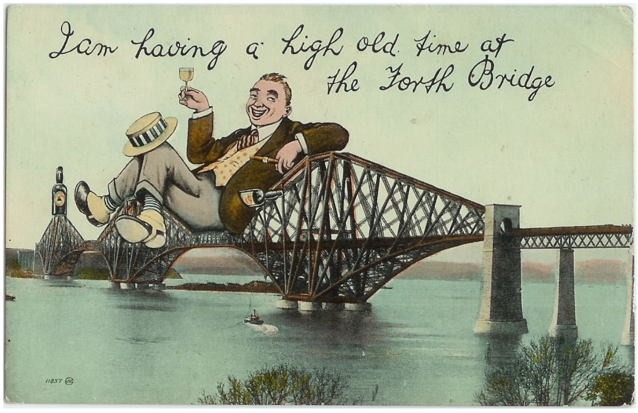
podjela sila prema području djelovanja:

- **koncentrirana sila**: djeluje u točki
- **distribuirana/raspodijeljena sila**: djeluje u svim točkama određenoga geometrijskog područja, dijela kontinuuma

  [kontinuum: matematički i fizički model prostora ili njegova dijela u kojemu se uzima da je prostor neprekinut, što znači da je beskrajno djeljiv („nešto, svaki dio čega ima dijelove” [C. S. Peirce]) i bez praznina ili prekida, pri čemu se svakoj točki mogu pridijeliti određena fizikalna svojstva]

distribuirane sile: linijske, plošne, zapreminske/volumenske



podjela sila prema dosegu:

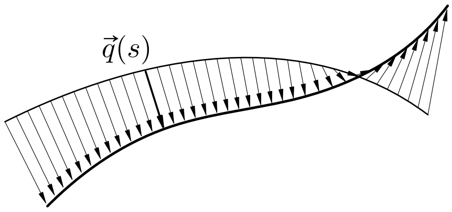
- **sile dalekoga dosega**: mogu se pojaviti između svakoga para čestica (primjerice, gravitacijska sila)

  djeluju na cijelo tijelo, pa se prikazuju zapreminskim silama
- **sile kratkoga dosega**: mogu se pojaviti samo između susjednih čestica

  kontaktne sile djeluju na dodirnoj površini tijelâ, a *unutarnje* preko zamišljene plohe kojom se razdvajaju dijelovi tijela, pa se prikazuju plošnim silama

matematički prikaz distribuiranih sila: vektorske funkcije jedne, dvije ili tri varijable:

$$\vec{q} \;:\; s \;\mapsto\; \vec{q}(s) \;=\; q_x(s)\,\vec{\imath} + q_y(s)\,\vec{\jmath} + q_z(s)\,\vec{k}, \quad s \in [s_1, s_2] \subseteq \mathbb{R}$$

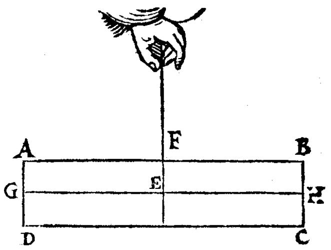


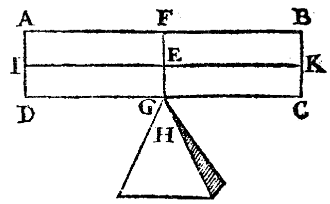
$$\vec{q} \;:\; (u,v) \;\mapsto\; \vec{q}(u,v) \;=\; q_x(u,v)\,\vec{\imath} + q_y(u,v)\,\vec{\jmath} + q_z(u,v)\,\vec{k}, \quad (u,v) \in A \subseteq \mathbb{R}^2$$

$$\vec{q} \;:\; (x,y,z) \;\mapsto\; \vec{q}(x,y,z) \;=\; q_x(x,y,z)\,\vec{\imath} + q_y(x,y,z)\,\vec{\jmath} + q_z(x,y,z)\,\vec{k}, \quad (x,y,z) \in V \subseteq \mathbb{R}^3$$

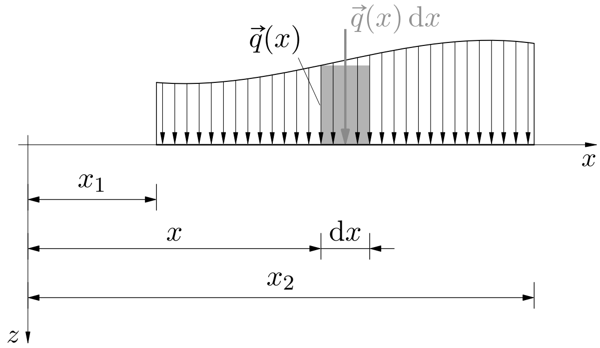
linijske sile

vertikalni pravac kroz težište pravokutnika jedna stranica kojega je horizontalna (iz Stevinovih *Počela umijeća vaganja* [1586.]):





sila na dijelu $[x_1, x_2]$ osi x, prikazana vektorskom funkcijom $\vec{q} \,:\, s \mapsto \vec{q}(x) = q(x)\,\vec{k}$:



- rezultirajuće djelovanje u ishodištu:
  - ◇ rezultirajuća sila:

$$\mathrm{d}F_R(x, \mathrm{d}x) \;=\; q(x)\,\mathrm{d}x$$

$$F_R \;=\; \int_{x_1}^{x_2} \mathrm{d}F_R(x, \mathrm{d}x) \;=\; \int_{x_1}^{x_2} q(x)\,\mathrm{d}x$$

$$\vec{F}_R \;=\; F_R\,\vec{k}$$

  - ◇ rezultirajući moment:

$$\mathrm{d}M_{R/0}(x, \mathrm{d}x) \;=\; \mathrm{d}F_R(x, \mathrm{d}x) \cdot \left(x + \frac{\mathrm{d}x}{2}\right) \;=\; q(x)\,x\,\mathrm{d}x \;+\; \underbrace{q(x)\,\frac{\mathrm{d}x^2}{2}}_{\text{zanemarivo}} \;=\; q(x)\,x\,\mathrm{d}x$$

$$M_{R/0} \;=\; \int_{x_1}^{x_2} \mathrm{d}M_{R/0}(x, \mathrm{d}x) \;=\; \int_{x_1}^{x_2} q(x)\,x\,\mathrm{d}x$$

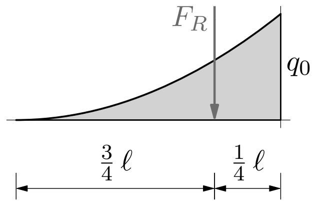
$$\vec{M}_{R/0} \;=\; -M_{R/0}\,\vec{\jmath}$$

- pravac djelovanja rezultante $\vec{F}_R \neq \vec{0}$:

$$x_R\,F_R \;=\; M_{R/0} \qquad \Longrightarrow \qquad x_R \;=\; \frac{M_{R/0}}{F_R} \;=\; \frac{\displaystyle\int_{x_1}^{x_2} q(x)\,x\,\mathrm{d}x}{\displaystyle\int_{x_1}^{x_2} q(x)\,\mathrm{d}x} \quad \text{ako je } \int_{x_1}^{x_2} q(x)\,\mathrm{d}x \;\neq\; 0$$

- primjeri:

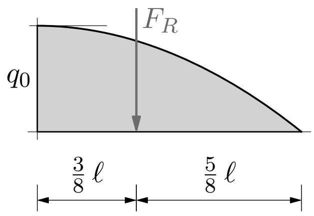
♠ $q(x) \;=\; \dfrac{q_0}{\ell^2}\,x^2$



$$F_R \;=\; \int_0^\ell \frac{q_0}{\ell^2}\,x^2\,\mathrm{d}x \;=\; \frac{q_0}{\ell^2}\int_0^\ell x^2\,\mathrm{d}x \;=\; \frac{q_0}{\ell^2}\,\frac{x^3}{3}\,\bigg|_0^\ell \;=\; \frac{q_0}{3\ell^2}\,\ell^3 \;=\; \frac{1}{3}\,q_0\,\ell$$

$$x_R \;=\; \frac{1}{F_R}\int_0^\ell \frac{q_0}{\ell^2}\,x^2\,x\,\mathrm{d}x \;=\; \frac{q_0}{\ell^2\,F_R}\int_0^\ell x^3\,\mathrm{d}x \;=\; \frac{q_0}{\ell^2\,F_R}\,\frac{x^4}{4}\,\bigg|_0^\ell \;=\; \frac{q_0}{4\,F_R}\,\ell^2 \;=\; \frac{3}{4}\,\ell$$

♠ $q(x) \;=\; q_0\left(1-\dfrac{x^2}{\ell^2}\right)$



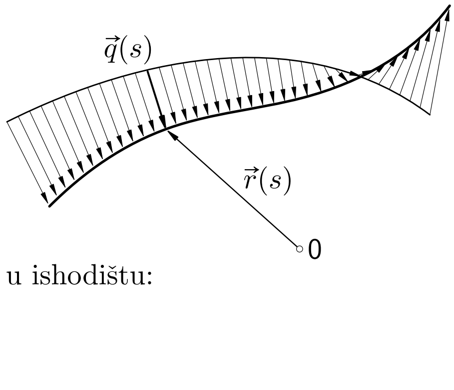
$$F_R \;=\; \int_0^\ell q_0\left(1-\frac{x^2}{\ell^2}\right)\mathrm{d}x \;=\; q_0\int_0^\ell \mathrm{d}x \;-\; \frac{q_0}{\ell^2}\int_0^\ell x^2\,\mathrm{d}x$$
$$\;=\; q_0\,x\,\bigg|_0^\ell \;-\; \frac{q_0}{\ell^2}\,\frac{x^3}{3}\,\bigg|_0^\ell \;=\; q_0\,\ell \;-\; \frac{1}{3}\,q_0\,\ell \;=\; \frac{2}{3}\,q_0\,\ell$$

$$x_R \;=\; \frac{1}{F_R}\int_0^\ell q_0\left(1-\frac{x^2}{\ell^2}\right)x\,\mathrm{d}x \;=\; \frac{q_0}{F_R}\left(\int_0^\ell x\,\mathrm{d}x \;-\; \frac{1}{\ell^2}\int_0^\ell x^3\,\mathrm{d}x\right)$$
$$\;=\; \frac{q_0}{F_R}\left(\frac{x^2}{2}\,\bigg|_0^\ell \;-\; \frac{x^4}{4\,\ell^2}\,\bigg|_0^\ell\right) \;=\; \frac{q_0}{4\,F_R}\,\ell^2 \;=\; \frac{3}{8}\,\ell$$

♠ $q(x) \;=\; \dfrac{q_0}{\ell}\,x,\;\; q(x) \;=\; q_0\left(1-\dfrac{x}{\ell}\right),\;\; q(x) \;=\; \dfrac{q_0}{\ell^3}\,x^3\;$ i $\;q(x) \;=\; q_0\left(1-\dfrac{x^3}{\ell^3}\right)$

… domaća zadaća!

opća sila $\vec{q}: s \mapsto \vec{q}(s) = \vec{q}_r\big(\vec{r}(s)\big)$ na dijelu prostorne krivulje $\vec{r}: s \mapsto \vec{r}(s)$ za $s \in [s_1, s_2] \subset \mathbb{R}$

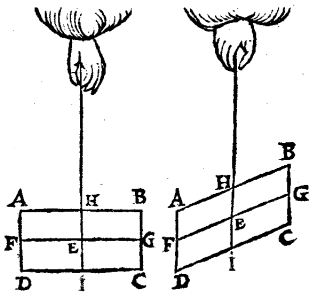


- rezultirajuće djelovanje u ishodištu:

$$\vec{F}_R \;=\; \int_{s_1}^{s_2} \vec{q}(s)\,\mathrm{d}s$$

$$\overrightarrow{M}_{R/0} \;=\; \int_{s_1}^{s_2} \vec{r}(s)\times\vec{q}(s)\,\mathrm{d}s$$

- poseban slučaj: $\vec{q}(s) = q(s)\,\vec{e}$



hvatište rezultante: $\vec{r}_R \;=\; \dfrac{\displaystyle\int_{s_1}^{s_2} q(s)\,\vec{r}(s)\,\mathrm{d}s}{\displaystyle\int_{s_1}^{s_2} q(s)\,\mathrm{d}s}$

uz uvjet $\displaystyle\int_{s_1}^{s_2} q(s)\,\mathrm{d}s \;\neq\; 0$

ako je $q > 0$ na $[s_1, s_2]$, riječ je o težištu dijela krivulje

plošne sile

sila $\vec{q} : (u,v) \mapsto \vec{q}(u,v) = \vec{q}_r\big(\vec{r}(u,v)\big)$ na liku ili dijelu plohe $\vec{r} : (u,v) \mapsto \vec{r}(u,v)$ za $(u,v) \in A \subset \mathbb{R}^2$

rezultirajuće djelovanje u ishodištu:

$$\vec{F}_R \;=\; \int_A \vec{q}(u,v)\,\mathrm{d}A$$

$$\overrightarrow{M}_{R/0} \;=\; \int_A \vec{r}(u,v) \times \vec{q}(u,v)\,\mathrm{d}A$$

zapreminske sile

sila $\vec{q} : (x,y,z) \mapsto \vec{q}(x,y,z)$ na tijelu V, $(x,y,z) \in V \subset \mathbb{R}^3$

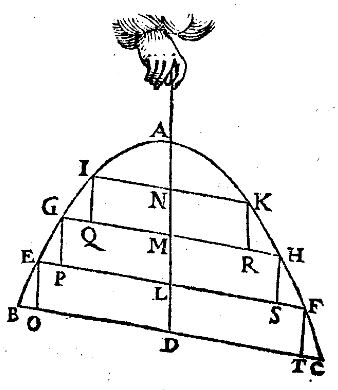
rezultirajuće djelovanje u ishodištu:

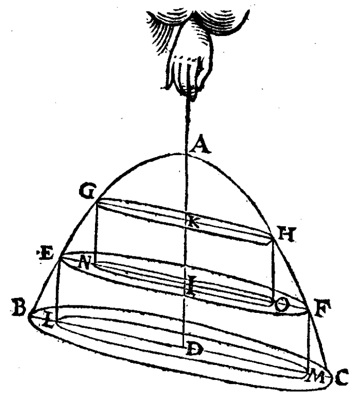
$$\vec{F}_R \;=\; \int_V \vec{q}(x,y,z)\,\mathrm{d}V$$

$$\overrightarrow{M}_{R/0} \;=\; \int_V \vec{r}(x,y,z) \times \vec{q}(x,y,z)\,\mathrm{d}V$$

težišta ravninskih likova, dijelova ploha i tijelâ ($q > 0$ na A ili V)

$$\vec{r}_T \;=\; \frac{\displaystyle\int_A q(u,v)\,\vec{r}(u,v)\,\mathrm{d}A}{\displaystyle\int_A q(u,v)\,\mathrm{d}A} \qquad \text{i} \qquad \vec{r}_T \;=\; \frac{\displaystyle\int_V q(x,y,z)\,\vec{r}(x,y,z)\,\mathrm{d}V}{\displaystyle\int_A q(x,y,z)\,\mathrm{d}V}$$





vektorski uvjeti ravnoteže tijela na koje djeluju koncentrirane i distribuirane sile i momenti:

$$\sum_{i=1}^{n} \vec{F}_i \;+\; \int_{s_1}^{s_2} \vec{q}_\ell(s)\,\mathrm{d}s \;+\; \int_A \vec{q}_p(u,v)\,\mathrm{d}A \;+\; \int_V \vec{q}_v(x,y,z)\,\mathrm{d}V \;=\; \vec{0}$$

$$\begin{aligned}\sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i \;+\; \sum_{j=1}^{m} \vec{M}_j \;+\; \int_{s_1}^{s_2} \vec{r}_\ell(s) \times \vec{q}_\ell(s)\,\mathrm{d}s \;+\; \int_{s_1}^{s_2} \vec{m}_\ell(s)\,\mathrm{d}s \\ +\; \int_A \vec{r}_p(u,v) \times \vec{q}_p(u,v)\,\mathrm{d}A \;+\; \int_A \vec{m}_p(u,v)\,\mathrm{d}A \\ +\; \int_V \vec{r}_v(x,y,z) \times \vec{q}_v(x,y,z)\,\mathrm{d}V \;=\; \vec{0}\end{aligned}$$

osnovna skalarna formulacija uvjetâ ravnoteže tijela na koje djeluju koncentrirane i distribuirane sile i momenti . . . domaća zadaća!