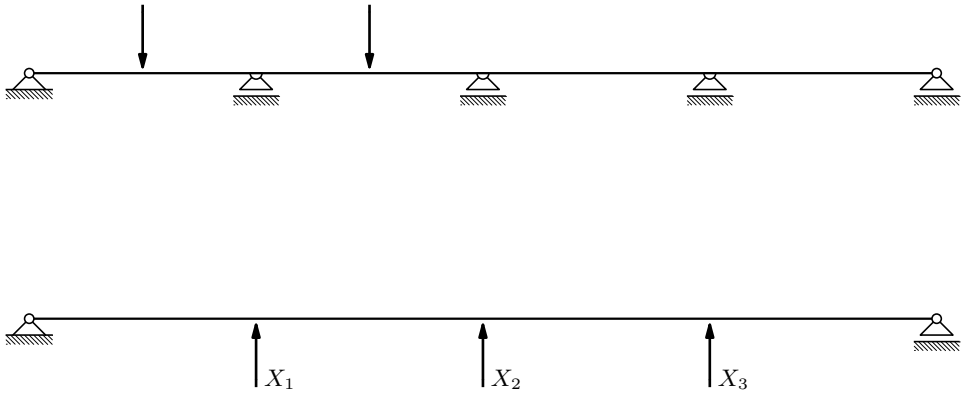
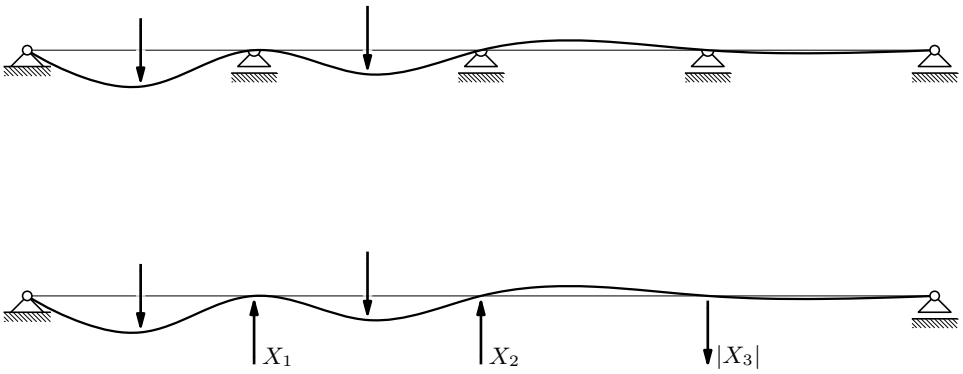
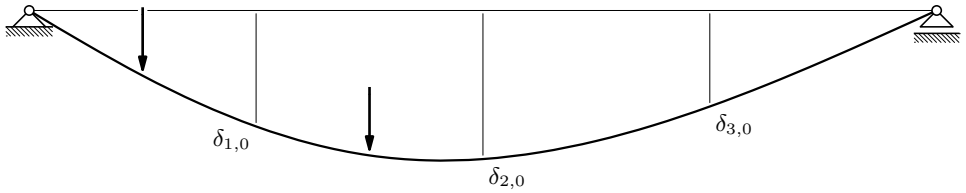
Građevna statika 2.

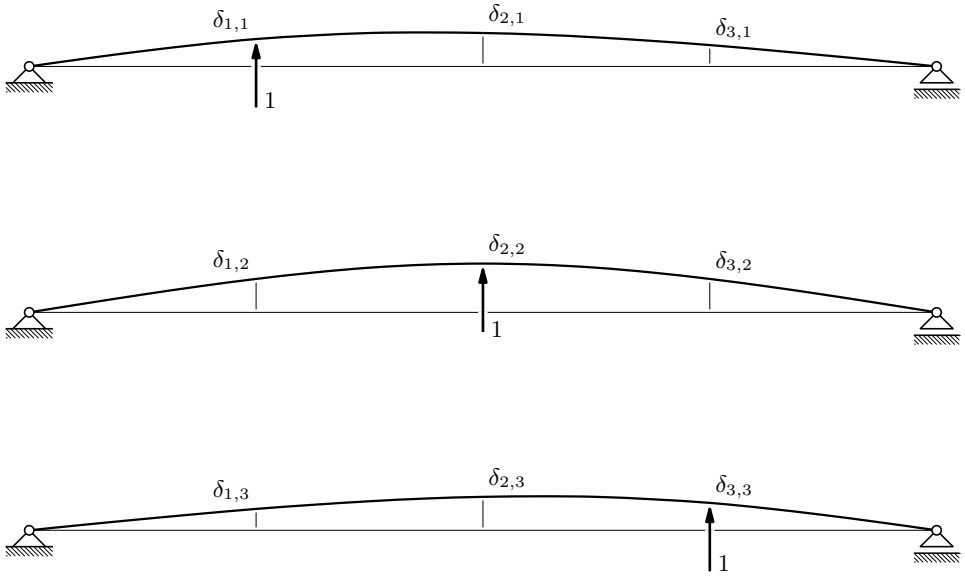
Rješavanje sustava linearnih jednadžbi

(Gaußov eliminacijski postupak)


X_1
X_2
X_3


X_1
X_2
$|X_3|$


$\delta_{1,0}$
$\delta_{2,0}$
$\delta_{3,0}$


$\delta_{1,1}$
$\delta_{2,1}$
$\delta_{3,1}$
1
$\delta_{1,2}$
$\delta_{2,2}$
$\delta_{3,2}$
1
$\delta_{1,3}$
$\delta_{2,3}$
$\delta_{3,3}$
1

$$\delta_{1,1} X_1 + \delta_{1,2} X_2 + \delta_{1,3} X_3 = -\delta_{1,0}$$

$$\delta_{2,1} X_1 + \delta_{2,2} X_2 + \delta_{2,3} X_3 = -\delta_{2,0}$$

$$\delta_{3,1} X_1 + \delta_{3,2} X_2 + \delta_{3,3} X_3 = -\delta_{3,0}$$

$$\delta_{1,1} X_1 + \delta_{1,2} X_2 + \delta_{1,3} X_3 = -\delta_{1,0} \quad \Big| \quad \times \frac{1}{\delta_{1,1}}$$

$$\delta_{2,1} X_1 + \delta_{2,2} X_2 + \delta_{2,3} X_3 = -\delta_{2,0}$$

$$\delta_{3,1} X_1 + \delta_{3,2} X_2 + \delta_{3,3} X_3 = -\delta_{3,0}$$

$$
\begin{aligned}
X_1 + \frac{\delta_{1,2}}{\delta_{1,1}} X_2 + \frac{\delta_{1,3}}{\delta_{1,1}} X_3 &= -\frac{\delta_{1,0}}{\delta_{1,1}} \quad \Big| \quad \times \ \delta_{2,1} \quad \Big| \quad (-) \\
\delta_{2,1} X_1 + \delta_{2,2} X_2 + \delta_{2,3} X_3 &= -\delta_{2,0} \\
\delta_{3,1} X_1 + \delta_{3,2} X_2 + \delta_{3,3} X_3 &= -\delta_{3,0}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \frac{\delta_{1,2}}{\delta_{1,1}} X_2 + \frac{\delta_{1,3}}{\delta_{1,1}} X_3 &= -\frac{\delta_{1,0}}{\delta_{1,1}} \\
\left(\delta_{2,2} - \delta_{2,1} \frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{2,3} - \delta_{2,1} \frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 &= -\delta_{2,0} + \delta_{2,1} \frac{\delta_{1,0}}{\delta_{1,1}} \\
\delta_{3,1} X_1 + \delta_{3,2} X_2 + \delta_{3,3} X_3 &= -\delta_{3,0}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \frac{\delta_{1,2}}{\delta_{1,1}} X_2 + \frac{\delta_{1,3}}{\delta_{1,1}} X_3 &= -\frac{\delta_{1,0}}{\delta_{1,1}} \quad \Big| \quad \times \delta_{3,1} \quad \Big| \searrow \\
\delta_{2,1} X_1 + \delta_{2,2} X_2 + \delta_{2,3} X_3 &= -\delta_{2,0} \qquad\qquad\qquad\quad (-) \\
\delta_{3,1} X_1 + \delta_{3,2} X_2 + \delta_{3,3} X_3 &= -\delta_{3,0} \qquad\qquad\qquad\quad \Big| \swarrow \\
&\qquad\qquad\qquad\qquad\qquad\quad \Big|
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \frac{\delta_{1,2}}{\delta_{1,1}} X_2 + \frac{\delta_{1,3}}{\delta_{1,1}} X_3 &= -\frac{\delta_{1,0}}{\delta_{1,1}} \\
\left(\delta_{2,2} - \delta_{2,1} \frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{2,3} - \delta_{2,1} \frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 &= -\delta_{2,0} + \delta_{2,1} \frac{\delta_{1,0}}{\delta_{1,1}} \\
\left(\delta_{3,2} - \delta_{3,1} \frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{3,3} - \delta_{3,1} \frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 &= -\delta_{3,0} + \delta_{3,1} \frac{\delta_{1,0}}{\delta_{1,1}}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
\tilde{\delta}_{2,2} X_2 + \tilde{\delta}_{2,3} X_3 &= -\tilde{\delta}_{2,0} \\
\tilde{\delta}_{3,2} X_2 + \tilde{\delta}_{3,3} X_3 &= -\tilde{\delta}_{3,0}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
\tilde{\delta}_{2,2} X_2 + \tilde{\delta}_{2,3} X_3 &= -\tilde{\delta}_{2,0} \quad \Big| \quad \times \frac{1}{\tilde{\delta}_{2,2}} \\
\tilde{\delta}_{3,2} X_2 + \tilde{\delta}_{3,3} X_3 &= -\tilde{\delta}_{3,0}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
X_2 + \frac{\tilde{\delta}_{2,3}}{\tilde{\delta}_{2,2}} X_3 &= -\frac{\tilde{\delta}_{2,0}}{\tilde{\delta}_{2,2}} \qquad \Big| \quad \times \tilde{\delta}_{3,2} \quad \Big| \quad (-) \\
\tilde{\delta}_{3,2} X_2 + \tilde{\delta}_{3,3} X_3 &= -\tilde{\delta}_{3,0}
\end{aligned}
$$

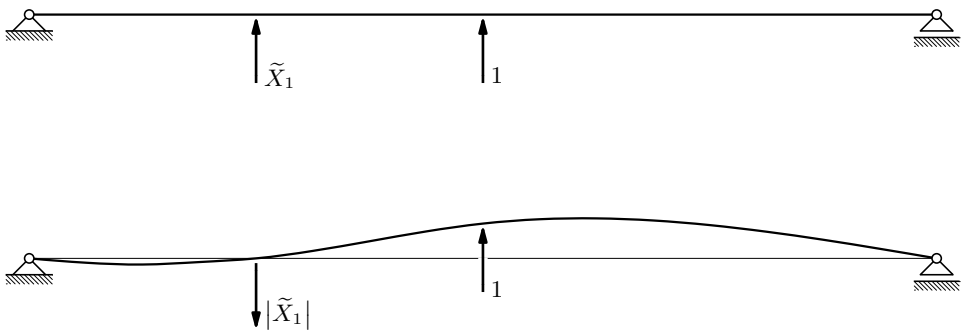
$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
X_2 + \frac{\tilde{\delta}_{2,3}}{\tilde{\delta}_{2,2}} X_3 &= -\frac{\tilde{\delta}_{2,0}}{\tilde{\delta}_{2,2}} \\
\left(\tilde{\delta}_{3,3} - \tilde{\delta}_{3,2} \frac{\tilde{\delta}_{2,3}}{\tilde{\delta}_{2,2}} \right) X_3 &= -\tilde{\delta}_{3,0} + \tilde{\delta}_{3,2} \frac{\tilde{\delta}_{2,0}}{\tilde{\delta}_{2,2}}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
X_2 + \tilde{\tilde{\delta}}_{2,3} X_3 &= -\tilde{\tilde{\delta}}_{2,0} \\
\tilde{\tilde{\delta}}_{3,3} X_3 &= -\tilde{\tilde{\delta}}_{3,0}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
X_2 + \tilde{\tilde{\delta}}_{2,3} X_3 &= -\tilde{\tilde{\delta}}_{2,0} \\
\tilde{\tilde{\delta}}_{3,3} X_3 &= -\tilde{\tilde{\delta}}_{3,0} \quad \Big| \quad \times \frac{1}{\tilde{\tilde{\delta}}_{3,3}}
\end{aligned}
$$

$$
\begin{aligned}
X_1 + \tilde{\delta}_{1,2} X_2 + \tilde{\delta}_{1,3} X_3 &= -\tilde{\delta}_{1,0} \\
X_2 + \tilde{\tilde{\delta}}_{2,3} X_3 &= -\tilde{\tilde{\delta}}_{2,0} \\
X_3 &= -\frac{\tilde{\tilde{\delta}}_{3,0}}{\tilde{\tilde{\delta}}_{3,3}}
\end{aligned}
$$

$$
\begin{aligned}
X_2 &= -\tilde{\tilde{\delta}}_{2,0} - \tilde{\tilde{\delta}}_{2,3} X_3 = -\tilde{\tilde{\delta}}_{2,0} - \tilde{\tilde{\delta}}_{2,3} \left(-\frac{\tilde{\tilde{\delta}}_{3,0}}{\tilde{\tilde{\delta}}_{3,3}} \right) \\
X_1 &= -\tilde{\delta}_{1,0} - \tilde{\delta}_{1,2} X_2 - \tilde{\delta}_{1,3} X_3 \\
&= -\tilde{\delta}_{1,0} - \tilde{\delta}_{1,2} \left(-\tilde{\tilde{\delta}}_{2,0} + \tilde{\tilde{\delta}}_{2,3} \frac{\tilde{\tilde{\delta}}_{3,0}}{\tilde{\tilde{\delta}}_{3,3}} \right) - \tilde{\delta}_{1,3} \left(-\frac{\tilde{\tilde{\delta}}_{3,0}}{\tilde{\tilde{\delta}}_{3,3}} \right)
\end{aligned}
$$



$$\widetilde{X}_1 \, \delta_{1,1} + \delta_{1,2} = 0$$

$$\widetilde{X}_1 = -\frac{\delta_{1,2}}{\delta_{1,1}}$$

$$\widetilde{X}_1 \, \delta_{2,1} + \delta_{2,2} = -\frac{\delta_{1,2}}{\delta_{1,1}} \, \delta_{2,1} + \delta_{2,2}$$

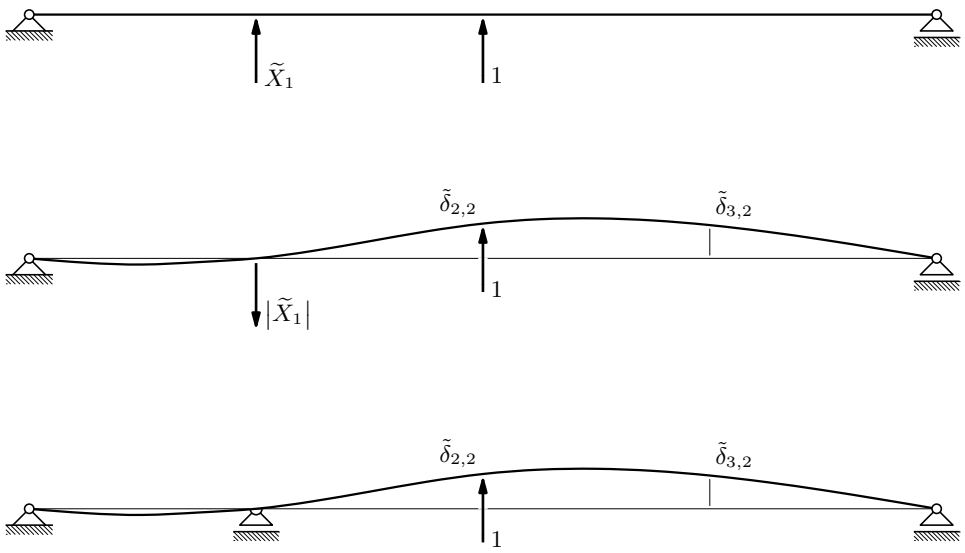
$$\widetilde{X}_1 \, \delta_{3,1} + \delta_{3,2} = -\frac{\delta_{1,2}}{\delta_{1,1}} \, \delta_{3,1} + \delta_{3,2}$$

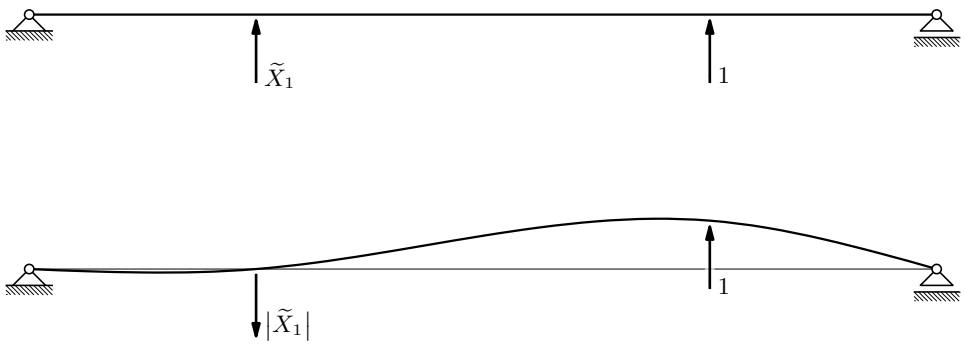
$$\tilde{X}_1\,\delta_{2,1} + \delta_{2,2} = -\frac{\delta_{1,2}}{\delta_{1,1}}\,\delta_{2,1} + \delta_{2,2} = \delta_{2,2} - \delta_{2,1}\,\frac{\delta_{1,2}}{\delta_{1,1}} = \tilde{\delta}_{2,2}$$

$$\tilde{X}_1\,\delta_{3,1} + \delta_{3,2} = -\frac{\delta_{1,2}}{\delta_{1,1}}\,\delta_{3,1} + \delta_{3,2} = \delta_{3,2} - \delta_{3,1}\,\frac{\delta_{1,2}}{\delta_{1,1}} = \tilde{\delta}_{3,2}$$

$$\left(\delta_{2,2} - \delta_{2,1}\,\frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{2,3} - \delta_{2,1}\,\frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 = -\delta_{2,0} + \delta_{2,1}\,\frac{\delta_{1,0}}{\delta_{1,1}}$$

$$\left(\delta_{3,2} - \delta_{3,1}\,\frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{3,3} - \delta_{3,1}\,\frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 = -\delta_{3,0} + \delta_{3,1}\,\frac{\delta_{1,0}}{\delta_{1,1}}$$


$\widetilde{X}_1$
1
$\tilde{\tilde{\delta}}_{2,2}$
$\tilde{\delta}_{3,2}$
1
$|\widetilde{X}_1|$
$\tilde{\tilde{\delta}}_{2,2}$
$\tilde{\delta}_{3,2}$
1



$$\widetilde{X}_1 \, \delta_{1,1} + \delta_{1,3} = 0$$

$$\widetilde{X}_1 = -\frac{\delta_{1,3}}{\delta_{1,1}}$$

$$\widetilde{X}_1 \, \delta_{2,1} + \delta_{2,3} = -\frac{\delta_{1,3}}{\delta_{1,1}} \, \delta_{2,1} + \delta_{2,3} = \delta_{2,3} - \delta_{2,1} \, \frac{\delta_{1,3}}{\delta_{1,1}} = \tilde{\delta}_{2,3}$$

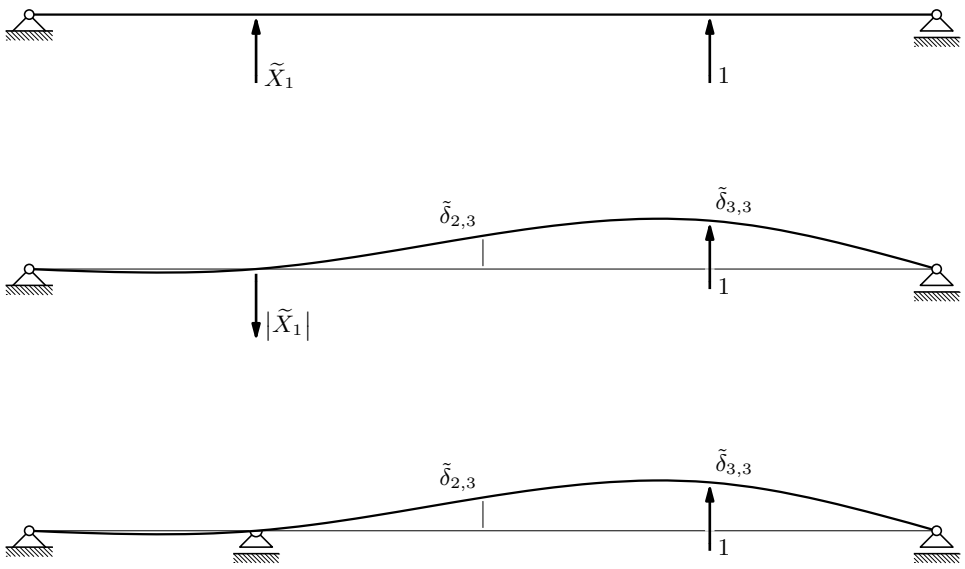
$$\widetilde{X}_1 \, \delta_{3,1} + \delta_{3,3} = -\frac{\delta_{1,3}}{\delta_{1,1}} \, \delta_{3,1} + \delta_{3,3} = \delta_{3,3} - \delta_{3,1} \, \frac{\delta_{1,3}}{\delta_{1,1}} = \tilde{\delta}_{3,3}$$

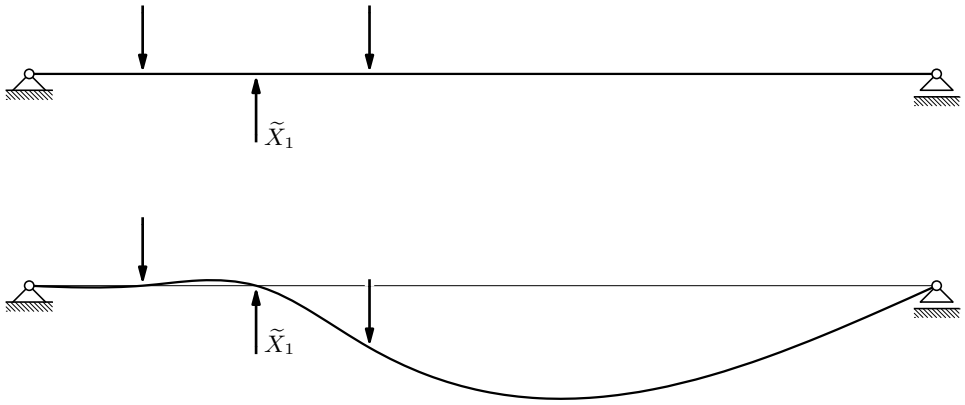
$$\tilde{X}_1\,\delta_{2,1} + \delta_{2,3} = -\frac{\delta_{1,3}}{\delta_{1,1}}\,\delta_{2,1} + \delta_{2,3} = \delta_{2,3} - \delta_{2,1}\,\frac{\delta_{1,3}}{\delta_{1,1}} = \tilde{\delta}_{2,3}$$

$$\tilde{X}_1\,\delta_{3,1} + \delta_{3,3} = -\frac{\delta_{1,3}}{\delta_{1,1}}\,\delta_{3,1} + \delta_{3,3} = \delta_{3,3} - \delta_{3,1}\,\frac{\delta_{1,3}}{\delta_{1,1}} = \tilde{\delta}_{3,3}$$

$$\left(\delta_{2,2} - \delta_{2,1}\,\frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{2,3} - \delta_{2,1}\,\frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 = -\delta_{2,0} + \delta_{2,1}\,\frac{\delta_{1,0}}{\delta_{1,1}}$$

$$\left(\delta_{3,2} - \delta_{3,1}\,\frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{3,3} - \delta_{3,1}\,\frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 = -\delta_{3,0} + \delta_{3,1}\,\frac{\delta_{1,0}}{\delta_{1,1}}$$


$\widetilde{X}_1$
1
$\tilde{\delta}_{2,3}$
$\tilde{\delta}_{3,3}$
1
$|\widetilde{X}_1|$
$\tilde{\delta}_{2,3}$
$\tilde{\delta}_{3,3}$
1



$$\widetilde{X}_1\,\delta_{1,1} + \delta_{1,0} = 0$$

$$\widetilde{X}_1 = -\frac{\delta_{1,0}}{\delta_{1,1}}$$

$$\widetilde{X}_1\,\delta_{2,1} + \delta_{2,0} = -\frac{\delta_{1,0}}{\delta_{1,1}}\,\delta_{2,1} + \delta_{2,0} = \delta_{2,0} - \delta_{2,1}\,\frac{\delta_{1,0}}{\delta_{1,1}} = \tilde{\delta}_{2,0}$$

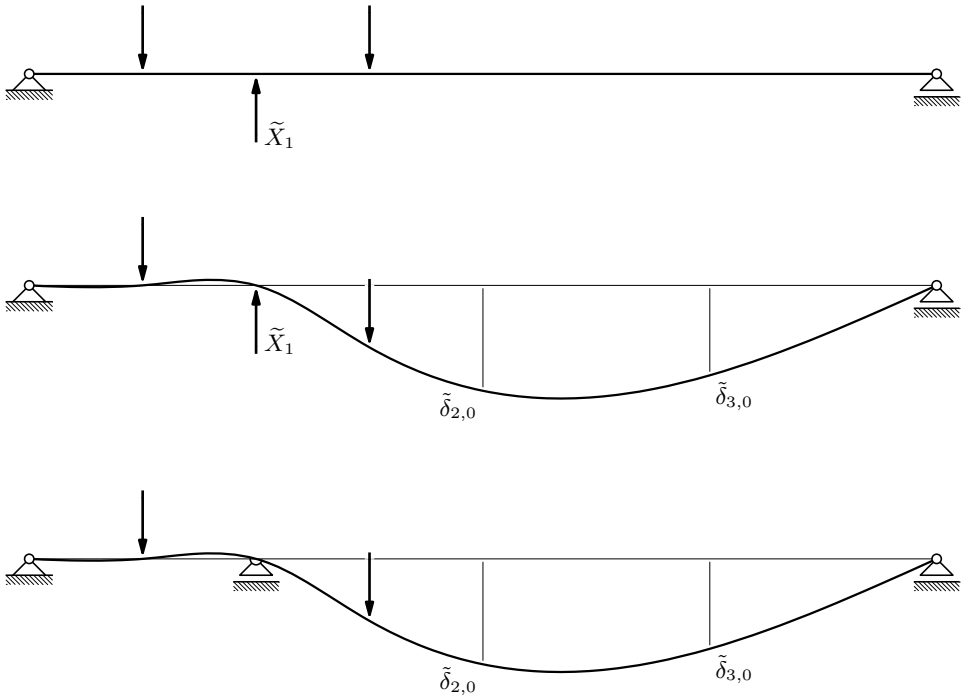
$$\widetilde{X}_1\,\delta_{3,1} + \delta_{3,0} = -\frac{\delta_{1,0}}{\delta_{1,1}}\,\delta_{3,1} + \delta_{3,0} = \delta_{3,0} - \delta_{3,1}\,\frac{\delta_{1,0}}{\delta_{1,1}} = \tilde{\delta}_{3,0}$$

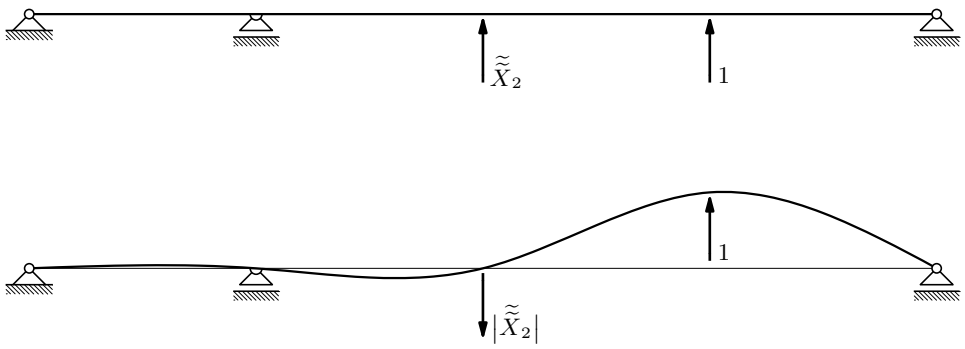
$$\tilde{X}_1\,\delta_{2,1} + \delta_{2,0} = -\frac{\delta_{1,0}}{\delta_{1,1}}\,\delta_{2,1} + \delta_{2,0} = \delta_{2,0} - \delta_{2,1}\,\frac{\delta_{1,0}}{\delta_{1,1}} = \tilde{\delta}_{2,0}$$

$$\tilde{X}_1\,\delta_{3,1} + \delta_{3,0} = -\frac{\delta_{1,0}}{\delta_{1,1}}\,\delta_{3,1} + \delta_{3,0} = \delta_{3,0} - \delta_{3,1}\,\frac{\delta_{1,0}}{\delta_{1,1}} = \tilde{\delta}_{3,0}$$

$$\left(\delta_{2,2} - \delta_{2,1}\,\frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{2,3} - \delta_{2,1}\,\frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 = -\delta_{2,0} + \delta_{2,1}\,\frac{\delta_{1,0}}{\delta_{1,1}}$$

$$\left(\delta_{3,2} - \delta_{3,1}\,\frac{\delta_{1,2}}{\delta_{1,1}}\right) X_2 + \left(\delta_{3,3} - \delta_{3,1}\,\frac{\delta_{1,3}}{\delta_{1,1}}\right) X_3 = -\delta_{3,0} + \delta_{3,1}\,\frac{\delta_{1,0}}{\delta_{1,1}}$$


$\widetilde{X}_1$
$\widetilde{X}_1$
$\tilde{\delta}_{2,0}$
$\tilde{\delta}_{3,0}$
$\tilde{\delta}_{2,0}$
$\tilde{\delta}_{3,0}$

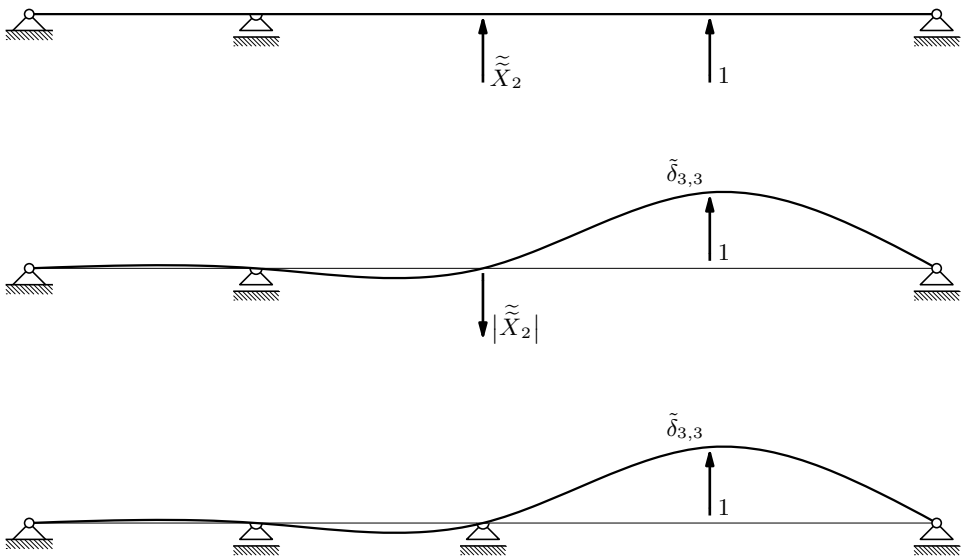


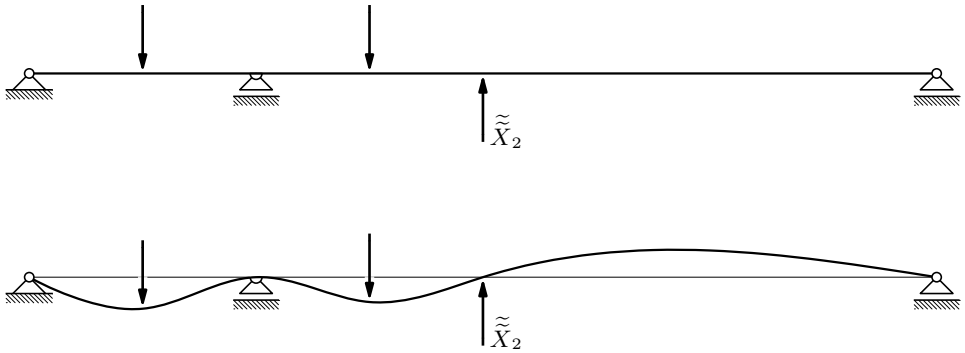
$$\tilde{\tilde{X}}_2\,\delta_{2,2} + \delta_{2,3} = 0$$

$$\tilde{\tilde{X}}_2 = -\frac{\delta_{2,3}}{\delta_{2,2}}$$

$$\tilde{\tilde{X}}_2\,\delta_{3,2} + \delta_{3,3} = -\frac{\delta_{2,3}}{\delta_{2,2}}\,\delta_{3,2} + \delta_{3,3} = \delta_{3,3} - \delta_{3,2}\,\frac{\delta_{2,3}}{\delta_{2,2}} = \tilde{\tilde{\delta}}_{3,3}$$

$$\left(\tilde{\delta}_{3,3} - \tilde{\delta}_{3,2}\,\frac{\tilde{\delta}_{2,3}}{\tilde{\delta}_{2,2}}\right) X_3 = -\tilde{\delta}_{3,0} + \tilde{\delta}_{3,2}\,\frac{\tilde{\delta}_{2,0}}{\tilde{\delta}_{2,2}}$$


$\tilde{\tilde{X}}_2$
1
$\tilde{\tilde{\delta}}_{3,3}$
1
$|\tilde{\tilde{X}}_2|$
$\tilde{\tilde{\delta}}_{3,3}$
1

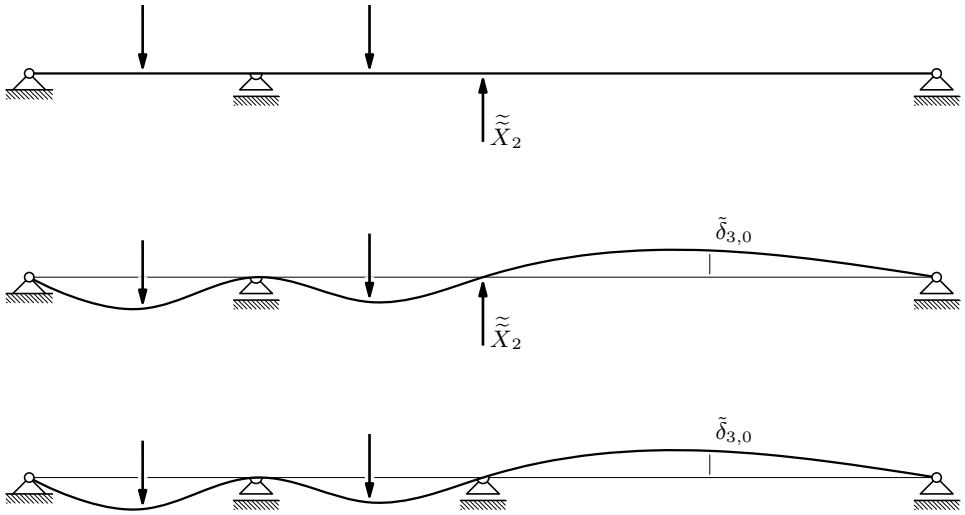


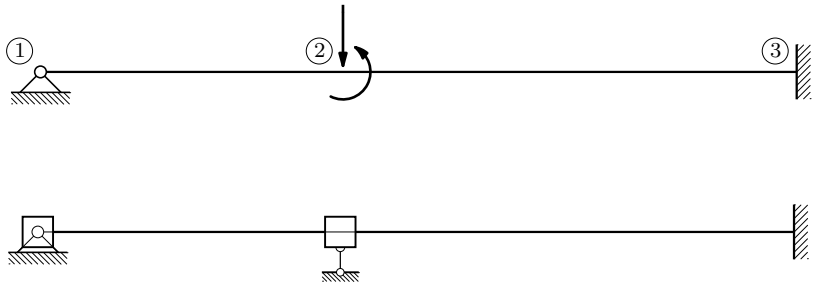
$$\tilde{\tilde{X}}_2\,\delta_{2,2} + \delta_{2,0} = 0$$

$$\tilde{\tilde{X}}_2 = -\frac{\delta_{2,0}}{\delta_{2,2}}$$

$$\tilde{\tilde{X}}_2\,\delta_{3,2} + \delta_{3,0} = -\frac{\delta_{2,0}}{\delta_{2,2}}\,\delta_{3,2} + \delta_{3,0} = \delta_{3,0} - \delta_{3,2}\,\frac{\delta_{2,0}}{\delta_{2,2}} = \tilde{\tilde{\delta}}_{3,0}$$

$$\left(\tilde{\delta}_{3,3} - \tilde{\delta}_{3,2}\,\frac{\tilde{\delta}_{2,3}}{\tilde{\delta}_{2,2}}\right) X_3 = -\tilde{\delta}_{3,0} + \tilde{\delta}_{3,2}\,\frac{\tilde{\delta}_{2,0}}{\tilde{\delta}_{2,2}}$$

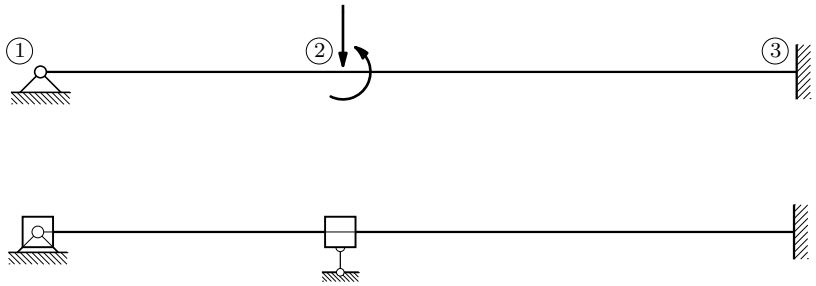

$\tilde{\tilde{X}}_2$
$\tilde{\delta}_{3,0}$
$\tilde{\tilde{X}}_2$
$\tilde{\delta}_{3,0}$



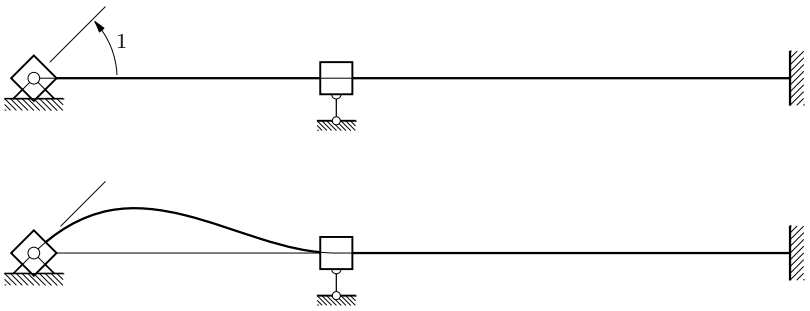
$$k_{1,1}\,\varphi_1 + k_{1,2}\,\varphi_2 + k_{1,3}\,w_2 = 0$$

$$k_{2,1}\,\varphi_1 + k_{2,2}\,\varphi_2 + k_{2,3}\,w_2 = M$$

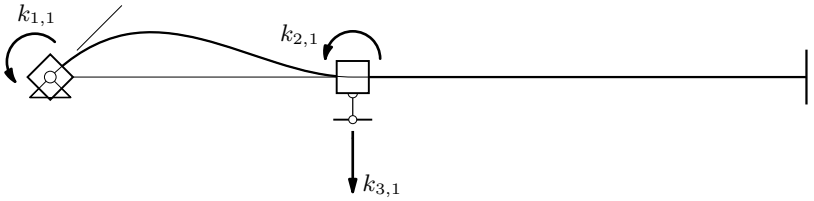
$$k_{3,1}\,\varphi_1 + k_{3,2}\,\varphi_2 + k_{3,3}\,w_2 = F$$

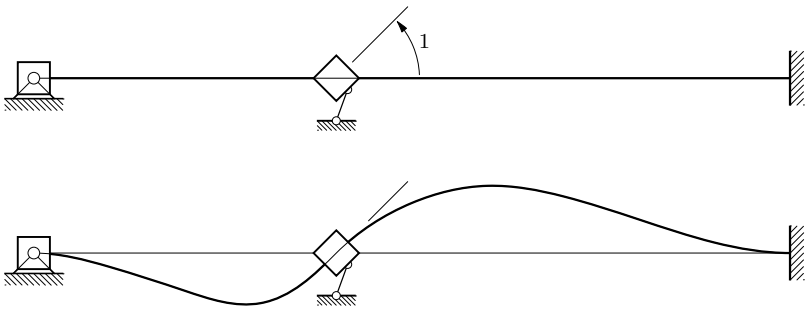


$$\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ M \\ F \end{bmatrix}$$

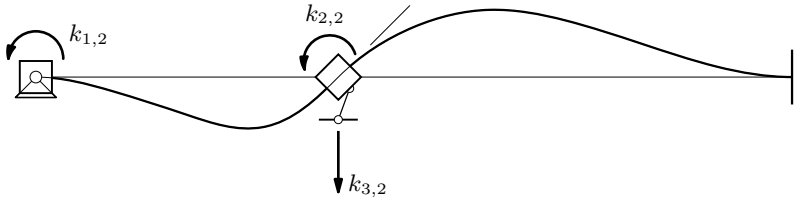


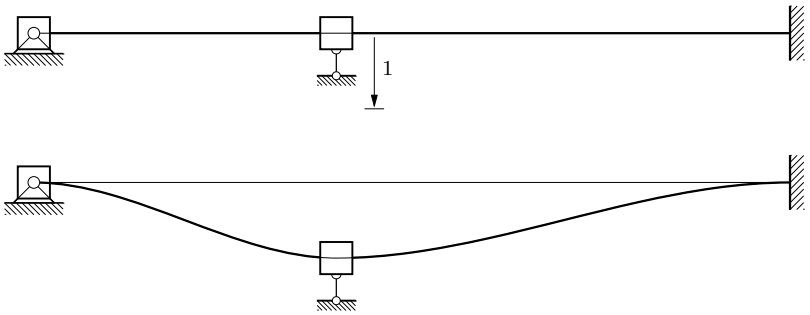
$$\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} k_{1,1} \\ k_{2,1} \\ k_{3,1} \end{bmatrix}$$



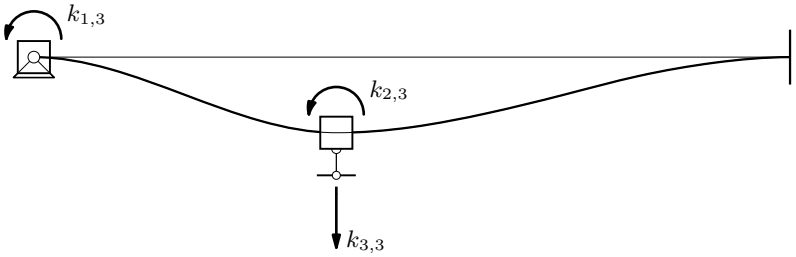


$$\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} k_{1,2} \\ k_{2,2} \\ k_{3,2} \end{bmatrix}$$





$$\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} k_{1,3} \\ k_{2,3} \\ k_{3,3} \end{bmatrix}$$

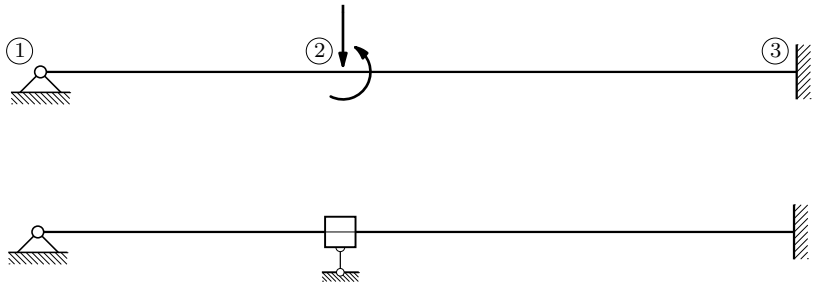


$$
\begin{aligned}
k_{1,1}\,\varphi_1 + k_{1,2}\,\varphi_2 \;+\; k_{1,3}\,w_2 \;&=\; 0 \\
\widetilde{k}_{2,2}\,\varphi_2 \;+\; \widetilde{k}_{2,3}\,w_2 \;&=\; M \\
\widetilde{k}_{3,2}\,\varphi_2 \;+\; \widetilde{k}_{3,3}\,w_2 \;&=\; F
\end{aligned}
$$

$$
\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ 0 & \widetilde{k}_{2,2} & \widetilde{k}_{2,3} \\ 0 & \widetilde{k}_{3,2} & \widetilde{k}_{3,3} \end{bmatrix}
\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ w_2 \end{bmatrix}
=
\begin{bmatrix} 0 \\ M \\ F \end{bmatrix}
$$

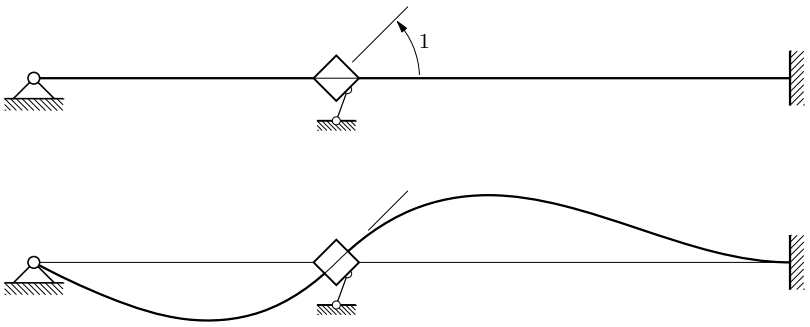
$$\widetilde{k}_{2,2}\,\varphi_2 \;+\; \widetilde{k}_{2,3}\,w_2 \;=\; M$$
$$\widetilde{k}_{3,2}\,\varphi_2 \;+\; \widetilde{k}_{3,3}\,w_2 \;=\; F$$

$$\begin{bmatrix} \widetilde{k}_{2,2} & \widetilde{k}_{2,3} \\ \widetilde{k}_{3,2} & \widetilde{k}_{3,3} \end{bmatrix} \begin{bmatrix} \varphi_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} M \\ F \end{bmatrix}$$

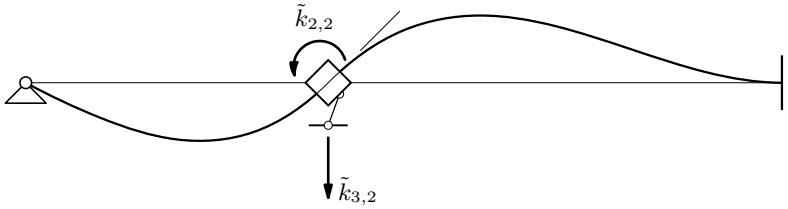


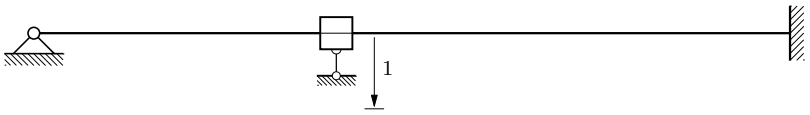
$$\tilde{k}_{2,2}\,\varphi_2 \;+\; \tilde{k}_{2,3}\,w_2 \;=\; M$$

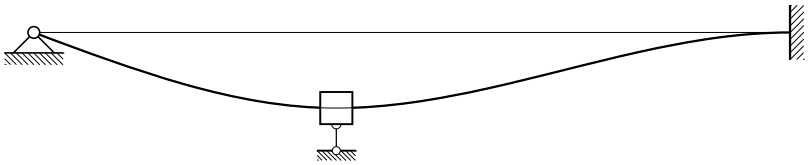
$$\tilde{k}_{3,2}\,\varphi_2 \;+\; \tilde{k}_{3,3}\,w_2 \;=\; F$$



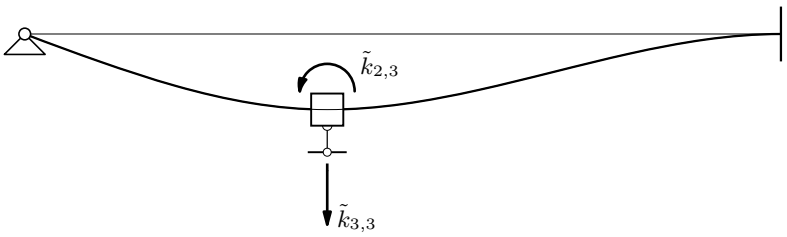
$$\begin{bmatrix} \widetilde{k}_{2,2} & \widetilde{k}_{2,3} \\ \widetilde{k}_{3,2} & \widetilde{k}_{3,3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \widetilde{k}_{2,2} \\ \widetilde{k}_{3,2} \end{bmatrix}$$





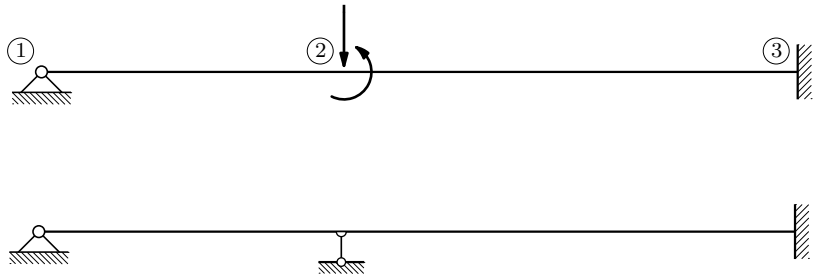


$$\begin{bmatrix} \tilde{k}_{2,2} & \tilde{k}_{2,3} \\ \tilde{k}_{3,2} & \tilde{k}_{3,3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \tilde{k}_{2,3} \\ \tilde{k}_{3,3} \end{bmatrix}$$

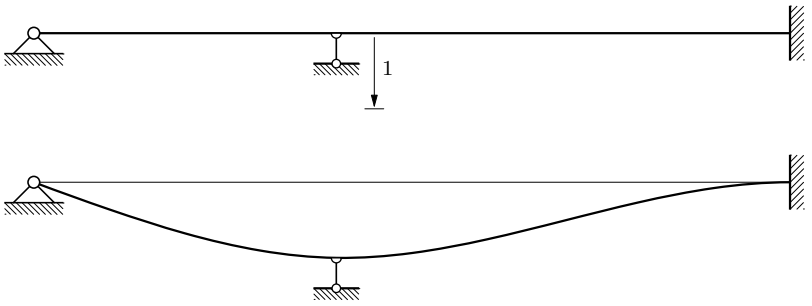
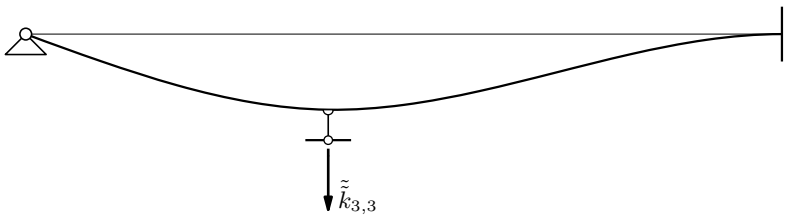
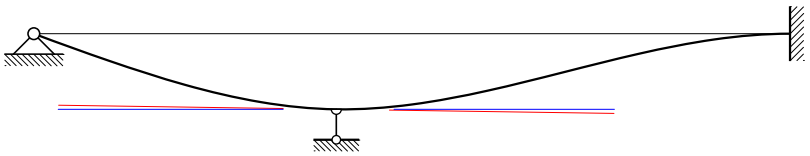


$$
\begin{aligned}
k_{1,1}\,\varphi_1 \;+\; k_{1,2}\,\varphi_2 + k_{1,3}\,w_2 &= 0 \\
\widetilde{k}_{2,2}\,\varphi_2 + \widetilde{k}_{2,3}\,w_2 &= M \\
\widetilde{\widetilde{k}}_{3,3}\,w_2 &= F - \kappa M
\end{aligned}
$$

$$
\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ 0 & \widetilde{k}_{2,2} & \widetilde{k}_{2,3} \\ 0 & 0 & \widetilde{\widetilde{k}}_{3,3} \end{bmatrix}
\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ w_2 \end{bmatrix}
=
\begin{bmatrix} 0 \\ M \\ F - \kappa M \end{bmatrix}
$$



$$\tilde{\tilde{k}}_{3,3}\, w_2 \;=\; F \;-\; \kappa\, M$$



$$\tilde{\tilde{k}}_{3,3} \cdot 1 = \tilde{\tilde{k}}_{3,3}$$