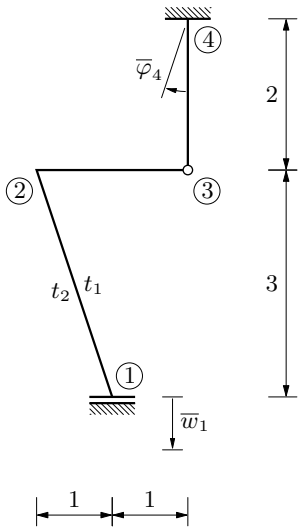
Inženjerska metoda pomakā

Primjer

Nacrtajte dijagram momenata savijanja!



$\overline{w}_1 \;=\; 3\ \text{mm}$

$\overline{\varphi}_4 \;=\; 0{,}000\,6$

$t_1 \;=\; 0^\circ\,\text{C}$

$t_2 \;=\; 42^\circ\,\text{C}$

$\alpha_t \;=\; 10^{-5}\,\text{K}^{-1}$

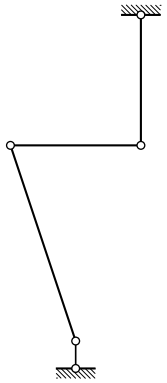
$EI \;=\; 162\,000\ \text{kNm}^2$

$(EI)_{\{2,3\}} \;=\; (EI)_{\{3,4\}} \;=\; EI$

$(EI)_{\{1,2\}} \;=\; 2\,EI$

$h_{\{1,2\}} \;=\; 60\ \text{cm}$

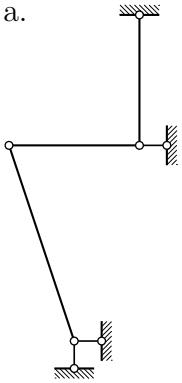
nepoznanice:



$S_{\text{min}} \;=\; 3 \cdot 2 \;-\; 4$

$\;\;\;=\; 2$

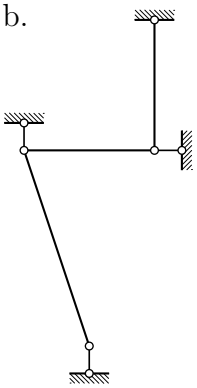
a.



φ_2

$u_1,\ u_{\{2,3\}}$

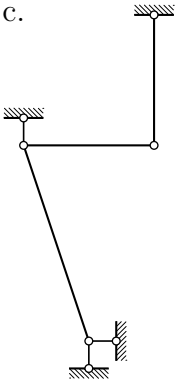
b.



φ_2

$w_2,\ u_{\{2,3\}}$

c.



φ_2

$u_1,\ w_2$

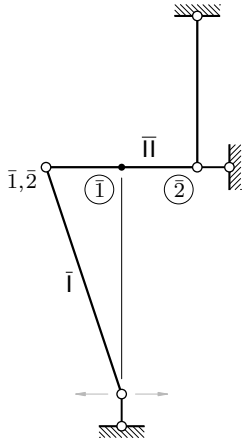
a.

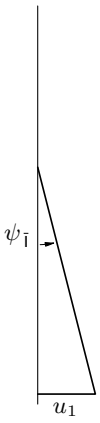
$$M_{1,2} \;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(u_1)} \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(u_{\{2,3\}})} \;+\; \overline{M}_{1,2}$$

$$M_{2,1} \;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(u_1)} \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(u_{\{2,3\}})} \;+\; \overline{M}_{2,1}$$

$$M_{2,3} \;=\; M_{2,3}^{\text{c}} \;=\; 3\,k_{\{2,3\}}\,\varphi_2 \;-\; 3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(u_1)} \;-\; 3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(u_{\{2,3\}})} \;+\; \overline{M}_{2,3}^{\text{c}}$$

$$M_{4,3} \;=\; M_{4,3}^{\text{c}} \;=\; -3\,k_{\{3,4\}}\,\psi_{\{3,4\}}^{(u_1)} \;-\; 3\,k_{\{3,4\}}\,\psi_{\{3,4\}}^{(u_{\{2,3\}})} \;+\; \overline{M}_{4,3}^{\text{c}}$$





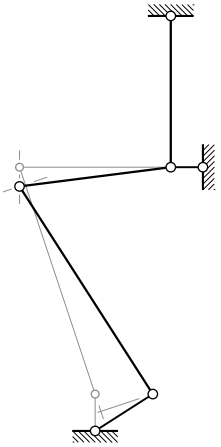
$$\psi^{(u_1)}_{\{1,2\}} \;=\; \psi_{\bar{\mathrm{I}}} \;=\; \frac{u_1}{3}$$

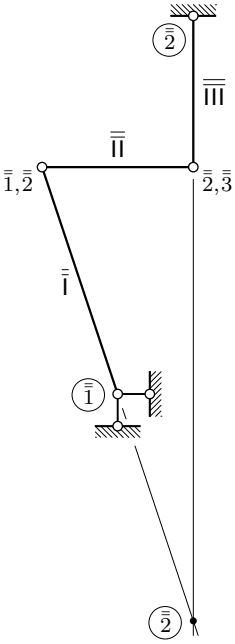
$$w_2^{(u_1)} \;=\; \psi_{\bar{\mathrm{I}}} \cdot 1$$

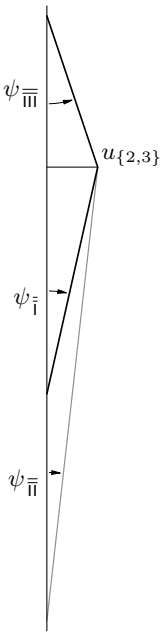
$$\psi^{(u_1)}_{\{2,3\}} \;=\; \psi_{\bar{\mathrm{II}}} \;=\; \frac{w_2^{(u_1)}}{2}$$

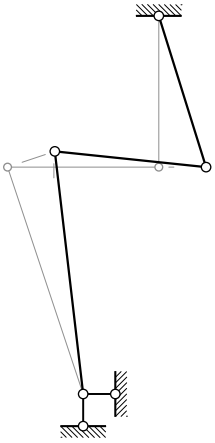
$$\;=\; \frac{\psi_{\bar{\mathrm{I}}}}{2} \;=\; \frac{u_1}{6}$$

$$\psi^{(u_1)}_{\{3,4\}} \;=\; 0$$









$$\psi^{(u_{\{2,3\}})}_{\{1,2\}} \;=\; \psi_{\bar{\bar{\mathrm{I}}}} \;=\; -\frac{u_{\{2,3\}}}{3}$$

$$\psi^{(u_{\{2,3\}})}_{\{2,3\}} \;=\; \psi_{\bar{\bar{\mathrm{II}}}} \;=\; -\frac{u_{\{2,3\}}}{6}$$

$$\psi^{(u_{\{2,3\}})}_{\{3,4\}} \;=\; \psi_{\bar{\bar{\mathrm{III}}}} \;=\; \frac{u_{\{2,3\}}}{2}$$

$$M_{1,2} \;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\frac{u_1}{3} \;-\; 6\,k_{\{1,2\}}\left(-\frac{u_{\{2,3\}}}{3}\right) \;+\; \overline{M}_{1,2}$$

$$\;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;-\; 2\,k_{\{1,2\}}\,u_1 \;+\; 2\,k_{\{1,2\}}\,u_{\{2,3\}} \;+\; \overline{M}_{1,2}$$

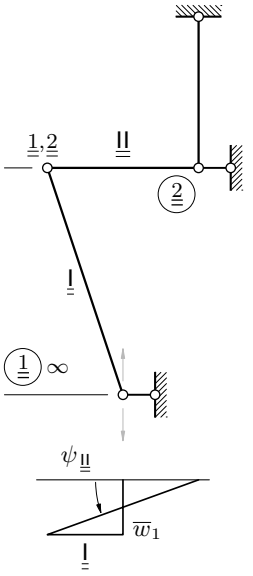
$$M_{2,1} \;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\frac{u_1}{3} \;-\; 6\,k_{\{1,2\}}\left(-\frac{u_{\{2,3\}}}{3}\right) \;+\; \overline{M}_{2,1}$$

$$\;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;-\; 2\,k_{\{1,2\}}\,u_1 \;+\; 2\,k_{\{1,2\}}\,u_{\{2,3\}} \;+\; \overline{M}_{2,1}$$

$$M_{2,3} \;=\; M^{\mathrm{c}}_{2,3} \;=\; 3\,k_{\{2,3\}}\,\varphi_2 \;-\; 3\,k_{\{2,3\}}\,\frac{u_1}{6} \;-\; 3\,k_{\{2,3\}}\left(-\frac{u_{\{2,3\}}}{6}\right) \;+\; \overline{M}^{\mathrm{c}}_{2,3}$$

$$\;=\; 3\,k_{\{2,3\}}\,\varphi_2 \;-\; \frac{1}{2}\,k_{\{2,3\}}\,u_1 \;+\; \frac{1}{2}\,k_{\{2,3\}}\,u_{\{2,3\}} \;+\; \overline{M}^{\mathrm{c}}_{2,3}$$

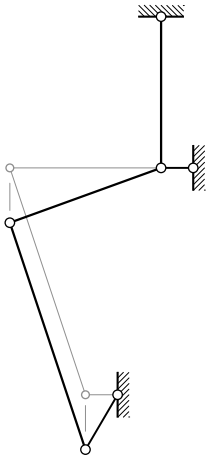
$$M_{4,3} \;=\; M^{\mathrm{c}}_{4,3} \;=\; -3\,k_{\{3,4\}}\cdot 0 \;-\; 3\,k_{\{3,4\}}\,\frac{u_{\{2,3\}}}{2} \;+\; \overline{M}^{\mathrm{c}}_{4,3} \;=\; -\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} \;+\; \overline{M}^{\mathrm{c}}_{4,3}$$

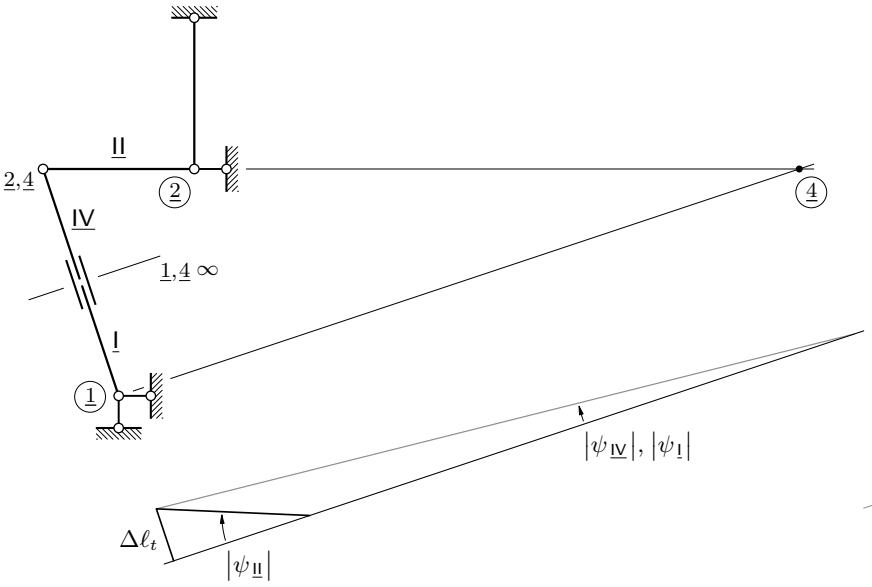


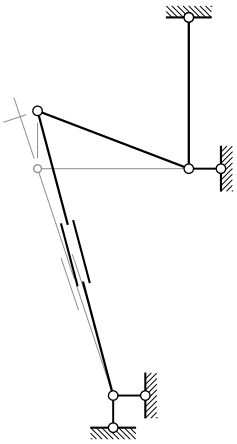
$$\psi_{\{1,2\}}^{(\overline{w}_1)} = \psi_{\underline{\underline{\mathrm{I}}}} = 0$$

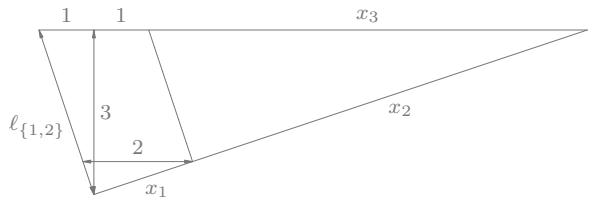
$$\psi_{\{2,3\}}^{(\overline{w}_1)} = \psi_{\underline{\underline{\mathrm{II}}}} = \frac{\overline{w}_1}{2} = \frac{0{,}003}{2} = 0{,}001\,5$$

$$\psi_{\{3,4\}}^{(\overline{w}_1)} = 0$$









$$\ell_{\{1,2\}} = \sqrt{1^2 + 9^2} = \sqrt{10} = 3{,}16 \text{ m}$$

$$t_\mathrm{s} = \frac{t_1 + t_2}{2} = \frac{0 + 42}{2} = 21^\circ \mathrm{C}$$

$$\Delta\ell_t = \alpha_t\, t_\mathrm{s}\, \ell_{\{1,2\}} = 10^{-5} \cdot 21 \cdot \sqrt{10} = 0{,}000\,664 \text{ m}$$

$$\frac{x_1}{2} = \frac{3}{\ell_{\{1,2\}}} \quad \Rightarrow \quad x_1 = \frac{6}{\ell_{\{1,2\}}} = \frac{6}{\sqrt{10}} = 1{,}90 \text{ m}$$

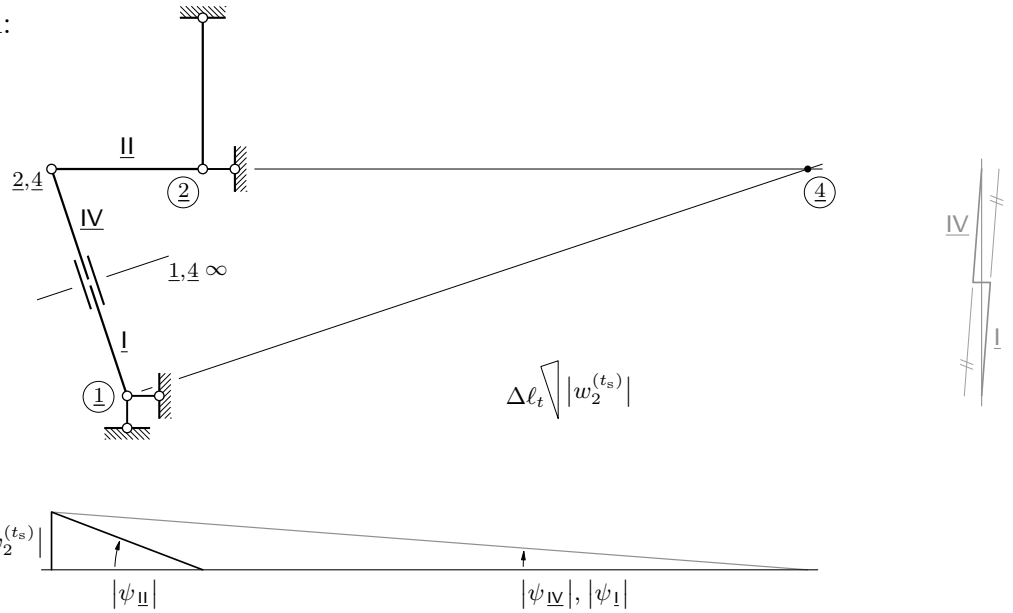
$$\frac{x_1 + x_2}{\ell_{\{1,2\}}} \;=\; \frac{3}{1} \qquad \Rightarrow \qquad x_1 + x_2 \;=\; 3\,\ell_{\{1,2\}} \;=\; 9{,}48 \text{ m}$$

$$\psi^{(t_\mathrm{s})}_{\{2,3\}} \;=\; \psi_{\underline{\mathrm{II}}} \;=\; -\frac{\Delta\ell_t}{x_1} \;=\; -\alpha_t\, t_\mathrm{s}\, \ell_{\{1,2\}}\, \frac{\ell_{\{1,2\}}}{6} \;=\; -10^{-5} \cdot 21 \cdot \frac{\left(\sqrt{10}\right)^2}{6} \;=\; -0{,}000\,350$$

$$\psi^{(t_\mathrm{s})}_{\{1,2\}} \;=\; \psi_{\underline{\mathrm{I}}} \;=\; -\frac{\Delta\ell_t}{x_1 + x_2} \;=\; -\frac{\alpha_t\, t_\mathrm{s}\, \ell_{\{1,2\}}}{3\,\ell_{\{1,2\}}} \;=\; -0{,}000\,070\,0$$

$$\psi^{(t_\mathrm{s})}_{\{3,4\}} \;=\; 0$$

ili:



objašnjenje trokut(ić)a sa stranicama $\Delta\ell_t$ i $\left|w_2^{(t_\mathrm{s})}\right|$:

$\overrightarrow{\Delta\ell}_t$ (vektor duljine $\Delta\ell_t$ na osi štapa $\{1,2\}$) nije pomak $\vec{\delta}_2$ čvora 2, tako da ni projekcije vektora $\overrightarrow{\Delta\ell}_t$ na vertikalnu i na horizontalnu os nisu projekcije pomaka $\vec{\delta}_2$ na te osi; $\overrightarrow{\Delta\ell}_t$ je samo komponenta pomaka $\vec{\delta}_2$ na osi štapa $\{1,2\}$, uzrokovana njegovim produljenjem. Osim što se produljuje, štap $\{1,2\}$ se zbog spoja sa štapom $\{2,3\}$ i zaokreće (oko pola $\underline{2}$), pa $\vec{\delta}_2$ ima i komponentu $\vec{w}_{1,2}$ okomitu na os štapa $\{1,2\}$. Kako čvor 2 „pripada" i štapu $\{2,3\}$, pomiče se po okomici na os toga štapa, tako da zbroj komponenata $\overrightarrow{\Delta\ell}_t$ i $\vec{w}_{1,2}$ pomaka $\vec{\delta}_2$ mora biti na toj okomici (pogledajte i plan pomakā na prethodnoj stranici), pa se $\vec{\delta}_2$ i njegova projekcija $\vec{w}_2$ na vertikalnu os (duljine $\left|w_2^{(t_\mathrm{s})}\right|$) poklapaju, $\vec{\delta}_2 = \vec{w}_2$, a projekcija $\vec{u}_2$ na horizontalnu os iščezava, $\vec{u}_2 = \vec{0}$, $u_2 = 0$.

$$\frac{\left|w_2^{(t_\mathrm{s})}\right|}{\Delta\ell_t} \;=\; \frac{\ell_{\{1,2\}}}{3} \qquad \Rightarrow \qquad \left|w_2^{(t_\mathrm{s})}\right| \;=\; \Delta\ell_t\, \frac{\ell_{\{1,2\}}}{3} \;=\; -10^{-5} \cdot 21 \cdot \frac{\left(\sqrt{10}\right)^2}{3} \;=\; 0{,}000\,700 \text{ m}$$

$$\psi^{(t_\mathrm{s})}_{\{2,3\}} \;=\; \psi_{\underline{\mathrm{II}}} \;=\; -\frac{\left|w_2^{(t_\mathrm{s})}\right|}{2} \;=\; -0{,}000\,350$$

$$\frac{2 + x_3}{\ell_{\{1,2\}}} \;=\; \frac{\ell_{\{1,2\}}}{1} \qquad \Rightarrow \qquad 2 + x_3 \;=\; \ell^2_{\{1,2\}} \;=\; 10 \text{ m}$$

(prema skici na prethodnoj stranici)

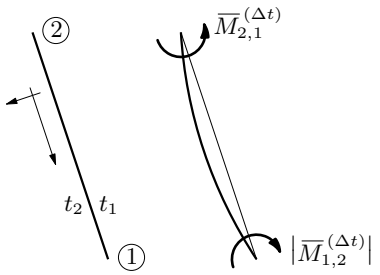
$$\psi^{(t_\mathrm{s})}_{\{1,2\}} \;=\; \psi_{\underline{\mathrm{I}}} \;=\; -\frac{\left|w_2^{(t_\mathrm{s})}\right|}{2 + x_3} \;=\; -0{,}000\,070\,0$$

$$\Delta t \;=\; t_{\text{dolje}} \,-\, t_{\text{gore}} \;=\; t_2 \,-\, t_1 \;=\; 42 \,-\, 0 \;=\; 42^\circ\,\text{C}$$

$$\overline{M}_{2,1}^{(\Delta t)} \;=\; (EI)_{\{1,2\}}\,\alpha_t\,\frac{\Delta_t}{h_{\{1,2\}}} \;=\; 2\,EI\,\alpha_t\,\frac{\Delta_t}{h_{\{1,2\}}}$$

$$\;=\; 2\cdot 162\,000\cdot 10^{-5}\cdot\frac{42}{0{,}6} \;=\; 226{,}80\ \text{kNm}$$

$$\overline{M}_{1,2}^{(\Delta t)} \;=\; -\overline{M}_{2,1}^{(\Delta t)} \;=\; -226{,}80\ \text{kNm}$$



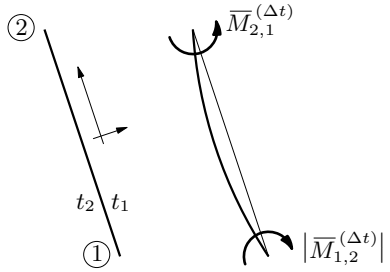
ili:

$$\Delta t \;=\; t_{\text{dolje}} \,-\, t_{\text{gore}} \;=\; t_1 \,-\, t_2 \;=\; 0 \,-\, 42 \;=\; -42^\circ\,\text{C}$$

$$\overline{M}_{1,2}^{(\Delta t)} \;=\; 2\,EI\,\alpha_t\,\frac{\Delta_t}{h_{\{1,2\}}}$$

$$\;=\; 2\cdot 162\,000\cdot 10^{-5}\cdot\frac{-42}{0{,}6} \;=\; -226{,}80\ \text{kNm}$$

$$\overline{M}_{2,1}^{(\Delta t)} \;=\; -\overline{M}_{1,2}^{(\Delta t)} \;=\; 226{,}80\ \text{kNm}$$



$${}^{\text{c}}\overline{M}_{4,3}^{(\overline{\varphi}_4)} \;=\; -3\,k_{\{3,4\}}\,\overline{\varphi}_4$$

$$k_{\{1,2\}} \;=\; \frac{2\,EI}{\ell_{\{1,2\}}} \;=\; \frac{2\cdot 162\,000}{\sqrt{10}} \;=\; 102\,458\ \text{kNm}$$

$$k_{\{2,3\}} \;=\; \frac{EI}{\ell_{\{2,3\}}} \;=\; \frac{162\,000}{2} \;=\; 81\,000\ \text{kNm}, \qquad k_{\{3,4\}} \;=\; \frac{EI}{\ell_{\{3,4\}}} \;=\; 81\,000\ \text{kNm}$$

$$\overline{M}_{1,2} \;=\; \overline{M}_{1,2}^{(\overline{w}_1)} \,+\, \overline{M}_{1,2}^{(t_{\text{s}})} \,+\, \overline{M}_{1,2}^{(\Delta t)} \;=\; -6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(\overline{w}_1)} \,-\, 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(t_{\text{s}})} \,+\, \overline{M}_{1,2}^{(\Delta t)}$$

$$\;=\; -6\cdot 102\,458\cdot 0 \,-\, 6\cdot 102\,458\cdot(-0{,}000\,07) \,-\, 226{,}80 \;=\; -183{,}77\ \text{kNm}$$

$$\overline{M}_{2,1} \;=\; \overline{M}_{2,1}^{(\overline{w}_1)} \,+\, \overline{M}_{2,1}^{(t_{\text{s}})} \,+\, \overline{M}_{1,2}^{(\Delta t)} \;=\; -6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(\overline{w}_1)} \,-\, 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(t_{\text{s}})} \,+\, \overline{M}_{1,2}^{(\Delta t)}$$

$$\;=\; -6\cdot 102\,458\cdot 0 \,-\, 6\cdot 102\,458\cdot(-0{,}000\,07) \,+\, 226{,}80 \;=\; 269{,}83\ \text{kNm}$$

$$\overline{M}_{2,3}^{\text{c}} \;=\; {}^{\text{c}}\overline{M}_{2,3}^{(\overline{w}_1)} \,+\, {}^{\text{c}}\overline{M}_{2,3}^{(t_{\text{s}})} \;=\; -3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(\overline{w}_1)} \,-\, 3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(t_{\text{s}})}$$

$$\;=\; -3\cdot 81\,000\cdot 0{,}001\,5 \,-\, 3\cdot 81\,000\cdot(-0{,}000\,35) \;=\; -279{,}45\ \text{kNm}$$

$$\overline{M}_{4,3}^{\text{c}} \;=\; {}^{\text{c}}\overline{M}_{4,3}^{(\overline{\varphi}_4)} \;=\; -3\,k_{\{3,4\}}\,\overline{\varphi}_4 \;=\; -3\cdot 81\,000\cdot(-0{,}000\,6) \;=\; -145{,}80\ \text{kNm}$$

jednadžba ravnoteže momenata u čvoru 2:

$$-\,M_{2,1} \,-\, M_{2,3} \;=\; 0 \qquad \Rightarrow \qquad M_{2,1} \,+\, M_{2,3} \;=\; 0$$

$$\left[4\,k_{\{1,2\}}\,\varphi_2 \;-\; 2\,k_{\{1,2\}}\,u_1 \;+\; 2\,k_{\{1,2\}}\,u_{\{2,3\}} \;+\; \overline{M}_{2,1}\right]$$
$$+\left[3\,k_{\{2,3\}}\,\varphi_2 \;-\; \frac{1}{2}\,k_{\{2,3\}}\,u_1 \;+\; \frac{1}{2}\,k_{\{2,3\}}\,u_{\{2,3\}} \;+\; \overline{M}^{\mathrm{c}}_{2,3}\right] \;=\; 0$$

$$(4\,k_{\{1,2\}} \;+\; 3\,k_{\{2,3\}})\,\varphi_2 \;+\; \left(-2\,k_{\{1,2\}} \;-\; \frac{1}{2}\,k_{\{2,3\}}\right) u_1$$
$$+\;\left(2\,k_{\{1,2\}} \;+\; \frac{1}{2}\,k_{\{2,3\}}\right) u_{\{2,3\}} \;=\; -\overline{M}_{2,1} \;-\; \overline{M}^{\mathrm{c}}_{2,3}$$

$$652\,832\cdot\varphi_2 \;-\; 245\,416\cdot u_1 \;+\; 245\,416\cdot u_{2,3} \;=\; -269{,}83 \;-\; (-279{,}45)$$

$$652\,832\cdot\varphi_2 \;-\; 245\,416\cdot u_1 \;+\; 245\,416\cdot u_{2,3} \;=\; 9{,}62 \qquad (\text{🦇})$$

jednadžba rada na virtualnom pomaku δu_1 (projekcije pomaka za u_1 uz dodani δ):

$$(M_{1,2} \;+\; M_{2,1})\,\delta\psi^{(\delta u_1)}_{\{1,2\}} \;+\; M_{2,3}\,\delta\psi^{(\delta u_1)}_{\{2,3\}} \;+\; M_{4,3}\,\delta\psi^{(\delta u_1)}_{\{3,4\}} \;=\; 0$$
$$\left(6\,k_{\{1,2\}}\,\varphi_2 \;-\; 4\,k_{\{1,2\}}\,u_1 \;+\; 4\,k_{\{1,2\}}\,u_{2,3} \;+\; \overline{M}_{1,2} \;+\; \overline{M}_{2,1}\right)\frac{\cancel{\delta u_1}}{3}$$
$$+\;\left(3\,k_{\{2,3\}}\,\varphi_2 \;-\; \frac{1}{2}\,k_{\{2,3\}}\,u_1 \;+\; \frac{1}{2}\,k_{\{2,3\}}\,u_{2,3} \;+\; \overline{M}^{\mathrm{c}}_{2,3}\right)\frac{\cancel{\delta u_1}}{6}$$
$$+\;\left(-\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} \;+\; \overline{M}^{\mathrm{c}}_{4,3}\right)\cdot 0 \;=\; 0 \qquad \forall\,\delta u_1$$

$$\left(2\,k_{\{1,2\}} \;+\; \frac{1}{2}\,k_{\{2,3\}}\right)\varphi_2 \;-\; \left(\frac{4}{3}\,k_{\{1,2\}} \;+\; \frac{1}{12}\,k_{\{2,3\}}\right) u_1$$
$$+\;\left(\frac{4}{3}\,k_{\{1,2\}} \;+\; \frac{1}{12}\,k_{\{2,3\}}\right) u_{2,3} \;=\; -\frac{1}{3}\,(\overline{M}_{1,2} \;+\; \overline{M}_{2,1}) \;-\; \frac{1}{6}\,\overline{M}^{\mathrm{c}}_{2,3}$$

$$245\,416\,\varphi_2 \;-\; 143\,361\,u_1 \;+\; 143\,361\,u_{2,3} \;=\; -\frac{1}{3}\cdot(-183{,}77 \;+\; 269{,}83) \;-\; \frac{1}{6}\cdot(-279{,}45)$$

$$245\,416\,\varphi_2 \;-\; 143\,361\,u_1 \;+\; 143\,361\,u_{2,3} \;=\; 17{,}89 \quad \Big| \quad \times(-1)$$

$$-245\,416\,\varphi_2 \;+\; 143\,361\,u_1 \;-\; 143\,361\,u_{2,3} \;=\; -17{,}89 \qquad (\text{🐉})$$

jednadžba rada na virtualnom pomaku $\delta u_{2,3}$ (projekcije pomaka za $u_{2,3}$ uz dodani δ):

$$(M_{1,2} \;+\; M_{2,1})\,\delta\psi^{(\delta u_{\{2,3\}})}_{\{1,2\}} \;+\; M_{2,3}\,\delta\psi^{(\delta u_{\{2,3\}})}_{\{2,3\}} \;+\; M_{4,3}\,\delta\psi^{(\delta u_{\{2,3\}})}_{\{3,4\}} \;=\; 0$$
$$\left(6\,k_{\{1,2\}}\,\varphi_2 \;-\; 4\,k_{\{1,2\}}\,u_1 \;+\; 4\,k_{\{1,2\}}\,u_{2,3} \;+\; \overline{M}_{1,2} \;+\; \overline{M}_{2,1}\right)\left(-\frac{\cancel{\delta u_{2,3}}}{3}\right)$$
$$+\;\left(3\,k_{\{2,3\}}\,\varphi_2 \;-\; \frac{1}{2}\,k_{\{2,3\}}\,u_1 \;+\; \frac{1}{2}\,k_{\{2,3\}}\,u_{2,3} \;+\; \overline{M}^{\mathrm{c}}_{2,3}\right)\left(-\frac{\cancel{\delta u_{2,3}}}{6}\right)$$
$$+\;\left(-\frac{3}{2}\,k_{\{3,4\}}\,u_{2,3} \;+\; \overline{M}^{\mathrm{c}}_{4,3}\right)\frac{\cancel{\delta u_{2,3}}}{2} \;=\; 0 \qquad \forall\,\delta u_1$$

$$-\left(2\,k_{\{1,2\}} \,+\, \frac{1}{2}\,k_{\{2,3\}}\right)\varphi_2 \,+\, \left(\frac{4}{3}\,k_{\{1,2\}} \,+\, \frac{1}{12}\,k_{\{2,3\}}\right)u_1$$
$$-\left(\frac{4}{3}\,k_{\{1,2\}} \,+\, \frac{1}{12}\,k_{\{2,3\}} \,+\, \frac{3}{4}\,k_{\{3,4\}}\right)u_{2,3}$$
$$= \frac{1}{3}\,(\overline{M}_{1,2} \,+\, \overline{M}_{2,1}) \,+\, \frac{1}{6}\,\overline{M}^{\mathrm{c}}_{2,3} \,-\, \frac{1}{2}\,\overline{M}^{\mathrm{c}}_{4,3}$$

$$-245\,416\,\varphi_2 \,+\, 143\,361\,u_1 \,-\, 204\,111\,u_{2,3}$$
$$= \frac{1}{3}\cdot(-183{,}77 \,+\, 269{,}83) \,+\, \frac{1}{6}\cdot(-279{,}45) \,-\, \frac{1}{2}\cdot(-145{,}80)$$

$$-245\,416\,\varphi_2 \,+\, 143\,361\,u_1 \,-\, 204\,111\,u_{2,3} \,=\, 55{,}01 \quad \Big| \quad \times(-1)$$

$$245\,416\,\varphi_2 \,-\, 143\,361\,u_1 \,+\, 204\,111\,u_{2,3} \,=\, -55{,}01 \qquad (\text{🐉})$$

rješenje sustava jednadžbi (🦇), (🐲) & (🐉):

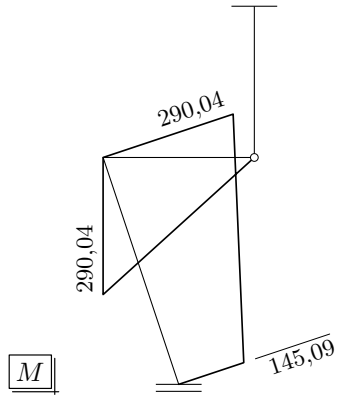
$$\varphi_2 \,=\, -0{,}000\,090\,26, \quad u_1 \,=\, -0{,}001\,479\ \mathrm{m} \quad \& \quad u_{\{2,3\}} \,=\, -0{,}001\,2\ \mathrm{m}$$

vrijednosti momenata na krajevima štapova:

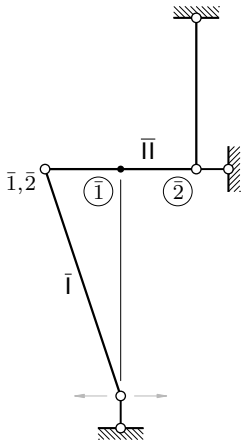
$$\begin{aligned}
M_{1,2} \,&=\, k_{\{1,2\}}\left(2\,\varphi_2 \,-\, 2\,u_1 \,+\, 2\,u_{\{2,3\}}\right) \,+\, \overline{M}_{1,2}\\
&=\, 102\,458\cdot\left[2\cdot(-0{,}000\,090\,26) \,-\, 2\cdot(-0{,}001\,479) \,+\, 2\cdot(-0{,}001\,2)\right] \,-\, 183{,}77\\
&=\, -145{,}09\ \mathrm{kNm}\\
M_{2,1} \,&=\, k_{\{1,2\}}\left(4\,\varphi_2 \,-\, 2\,u_1 \,+\, 2\,u_{\{2,3\}}\right) \,+\, \overline{M}_{2,1}\\
&=\, 102\,458\cdot\left[4\cdot(-0{,}000\,090\,26) \,-\, 2\cdot(-0{,}001\,479) \,+\, 2\cdot(-0{,}001\,2)\right] \,+\, 269{,}83\\
&=\, 290{,}01\ \mathrm{kNm}\\
M_{2,3} \,&=\, k_{\{2,3\}}\left(3\,\varphi_2 \,-\, \frac{1}{2}\,u_1 \,+\, \frac{1}{2}\,u_{\{2,3\}}\right) \,+\, \overline{M}^{\mathrm{c}}_{2,3}\\
&=\, 81\,000\cdot\left[3\cdot(-0{,}000\,090\,26) \,-\, \frac{1}{2}\cdot(-0{,}001\,479) \,+\, \frac{1}{2}\cdot(-0{,}001\,2)\right] \,-\, 279{,}45\\
&=\, -290{,}08\ \mathrm{kNm}\\
M_{4,3} \,&=\, -\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} \,+\, \overline{M}^{\mathrm{c}}_{4,3} \,=\, -\frac{3}{2}\cdot 81\,000\cdot(-0{,}001\,2) \,-\, 145{,}80 \,=\, 0
\end{aligned}$$

(„neravnoteža” momenata u čvoru 2 posljedica je pogrešaka zaokruživanja)

momentni dijagram:



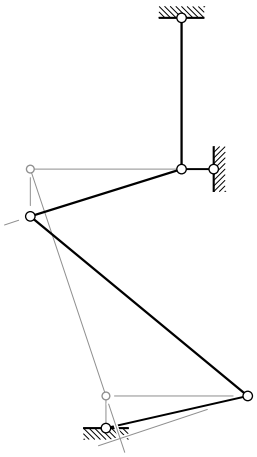
b.

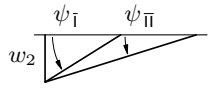


$$\psi^{(w_2)}_{\{1,2\}} = \psi_{\bar{\mathrm{I}}} = \frac{w_2}{1}$$

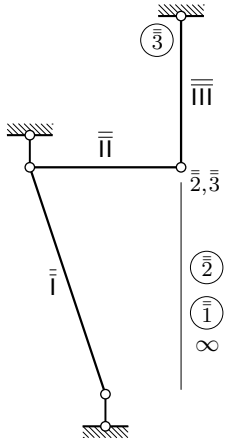
$$\psi^{(w_2)}_{\{2,3\}} = \psi_{\bar{\mathrm{II}}} = \frac{w_2}{2}$$

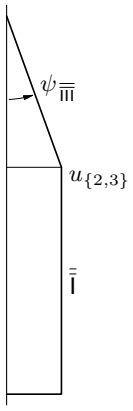
$$\psi^{(w_2)}_{\{3,4\}} = 0$$





(isti mehanizam kao za $\vec{u}_1$ u a., ali je „vodeći" pomak sada $\vec{w}_2$)

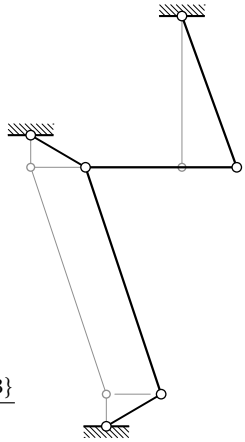




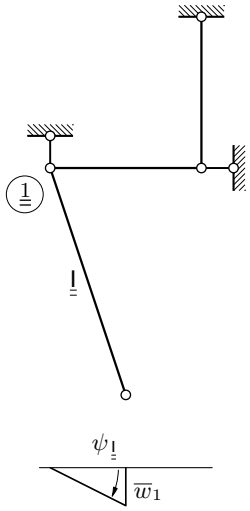
$$\psi^{(u_{\{2,3\}})}_{\{1,2\}} = 0$$

$$\psi^{(u_{\{2,3\}})}_{\{2,3\}} = 0$$

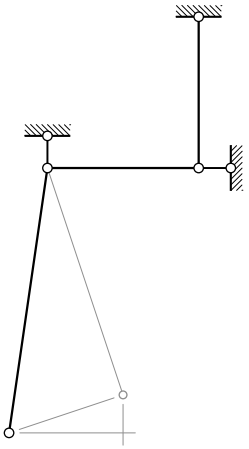
$$\psi^{(u_{\{2,3\}})}_{\{3,4\}} = \psi_{\bar{\bar{\mathrm{III}}}} = \frac{u_{\{2,3\}}}{2}$$

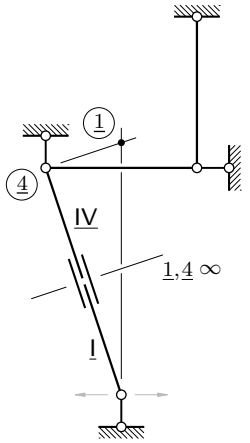


$$
\begin{aligned}
M_{1,2} &= 2\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(w_2)} - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(u_{\{2,3\}})} + \overline{M}_{1,2} \\
&= 2\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,w_2 + \overline{M}_{1,2} \\
M_{2,1} &= 4\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(w_2)} - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(u_{\{2,3\}})} + \overline{M}_{2,1} \\
&= 4\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,w_2 + \overline{M}_{2,1} \\
M_{2,3} &= M_{2,3}^{\mathrm{c}} = 3\,k_{\{2,3\}}\,\varphi_2 - 3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(w_2)} - 3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(u_{\{2,3\}})} + \overline{M}_{2,3}^{\mathrm{c}} \\
&= 3\,k_{\{2,3\}}\,\varphi_2 - \frac{3}{2}\,k_{\{2,3\}}\,w_2 + \overline{M}_{2,3}^{\mathrm{c}} \\
M_{4,3} &= M_{4,3}^{\mathrm{c}} = -3\,k_{\{3,4\}}\,\psi_{\{3,4\}}^{(w_2)} - 3\,k_{\{3,4\}}\,\psi_{\{3,4\}}^{(u_{\{2,3\}})} + \overline{M}_{4,3}^{\mathrm{c}} \\
&= -\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} + \overline{M}_{4,3}^{\mathrm{c}}
\end{aligned}
$$



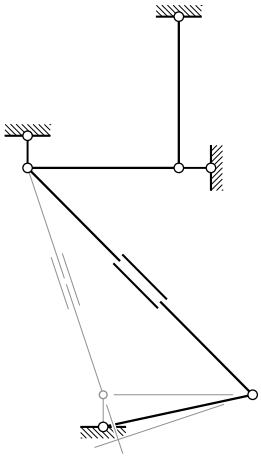
$$
\begin{aligned}
\psi_{\{1,2\}}^{(\overline{w}_1)} &= \psi_{\underline{\mathrm{I}}} = -\frac{\overline{w}_1}{1} \\
&= -0{,}003 \\
\psi_{\{2,3\}}^{(\overline{w}_1)} &= \psi_{\{3,4\}}^{(\overline{w}_1)} = 0
\end{aligned}
$$

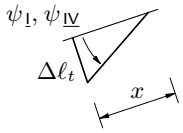




$$
\frac{x}{1} = \frac{\ell_{\{1,2\}}}{3} \quad \Rightarrow \quad x = \frac{\ell_{\{1,2\}}}{3}
$$

$$
\begin{aligned}
\psi_{\{1,2\}}^{(t_{\mathrm{s}})} &= \psi_{\underline{\mathrm{I}}} = \frac{\Delta\ell_t}{x} \\
&= \frac{3\,\Delta\ell_t}{\ell_{\{1,2\}}} = 0{,}000\,630 \\
\psi_{\{2,3\}}^{(t_{\mathrm{s}})} &= \psi_{\{3,4\}}^{(t_{\mathrm{s}})} = 0
\end{aligned}
$$





$$\overline{M}_{1,2} = \overline{M}_{1,2}^{(\overline{w}_1)} + \overline{M}_{1,2}^{(t_\mathrm{s})} + \overline{M}_{1,2}^{(\Delta t)} = -6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(\overline{w}_1)} - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(t_\mathrm{s})} + \overline{M}_{1,2}^{(\Delta t)}$$
$$= -6 \cdot 102\,458 \cdot (-0{,}003) - 6 \cdot 102\,458 \cdot 0{,}000\,63 - 226{,}80 = 1\,230{,}15\ \mathrm{kNm}$$
$$\overline{M}_{2,1} = \overline{M}_{2,1}^{(\overline{w}_1)} + \overline{M}_{2,1}^{(t_\mathrm{s})} + \overline{M}_{2,1}^{(\Delta t)} = -6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(\overline{w}_1)} - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}^{(t_\mathrm{s})} + \overline{M}_{2,1}^{(\Delta t)}$$
$$= -6 \cdot 102\,458 \cdot (-0{,}003) - 6 \cdot 102\,458 \cdot 0{,}000\,63 + 226{,}80 = 1\,683{,}75\ \mathrm{kNm}$$
$$\overline{M}_{2,3}^{\mathrm{c}} = {}^{\mathrm{c}}\overline{M}_{2,3}^{(\overline{w}_1)} + {}^{\mathrm{c}}\overline{M}_{2,3}^{(t_\mathrm{s})} = -3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(\overline{w}_1)} - 3\,k_{\{2,3\}}\,\psi_{\{2,3\}}^{(t_\mathrm{s})} = 0$$
$$\overline{M}_{4,3}^{\mathrm{c}} = {}^{\mathrm{c}}\overline{M}_{4,3}^{(\overline{\varphi}_4)} = -3\,k_{\{3,4\}}\,\overline{\varphi}_4 = -3 \cdot 81\,000 \cdot (-0{,}000\,6) = -145{,}80\ \mathrm{kNm}$$

jednadžba ravnoteže momenata u čvoru 2:

$$-M_{2,1} - M_{2,3} = 0 \qquad \Rightarrow \qquad M_{2,1} + M_{2,3} = 0$$
$$\left[4\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,w_2 + \overline{M}_{2,1}\right] + \left[3\,k_{\{2,3\}}\,\varphi_2 - \frac{3}{2}\,k_{\{2,3\}}\,w_2 + \overline{M}_{2,3}^{\mathrm{c}}\right] = 0$$
$$\left(4\,k_{\{1,2\}} + 3\,k_{\{2,3\}}\right)\varphi_2 + \left(-6\,k_{\{1,2\}} - \frac{3}{2}\,k_{\{2,3\}}\right) w_2 = -\overline{M}_{2,1} - \overline{M}_{2,3}^{\mathrm{c}}$$
$$652\,832 \cdot \varphi_2 - 736\,248 \cdot w_2 = -1\,683{,}75 \qquad (\text{}_{\mathrm{b}})$$

jednadžba rada na virtualnom pomaku δw_2:

$$\left(M_{1,2} + M_{2,1}\right)\delta\psi_{\{1,2\}}^{(\delta w_2)} + M_{2,3}\,\delta\psi_{\{2,3\}}^{(\delta w_2)} + M_{4,3}\,\delta\psi_{\{3,4\}}^{(\delta w_2)} = 0$$
$$\left(6\,k_{\{1,2\}}\,\varphi_2 - 12\,k_{\{1,2\}}\,w_2 + \overline{M}_{1,2} + \overline{M}_{2,1}\right)\cancel{\delta w_2}$$
$$+ \left(3\,k_{\{2,3\}}\,\varphi_2 - \frac{3}{2}\,k_{\{2,3\}}\,w_2 + \overline{M}_{2,3}^{\mathrm{c}}\right)\frac{\cancel{\delta w_2}}{2} + \left(-\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} + \overline{M}_{4,3}^{\mathrm{c}}\right) \cdot 0 = 0$$
$$\left(6\,k_{\{1,2\}} + \frac{3}{2}\,k_{\{2,3\}}\right)\varphi_2 - \left(12\,k_{\{1,2\}} + \frac{3}{4}\,k_{\{2,3\}}\right) w_2 = -\overline{M}_{1,2} - \overline{M}_{2,1} - \frac{1}{2}\,\overline{M}_{2,3}^{\mathrm{c}}$$
$$736\,248\,\varphi_2 - 1\,290\,246\,w_2 = -1\,230{,}15 - 1\,683{,}75 - \frac{1}{2} \cdot 0$$
$$736\,248\,\varphi_2 - 1\,290\,246\,w_2 = -2\,913{,}90 \quad \Big| \quad \times (-1)$$
$$-736\,248\,\varphi_2 + 1\,290\,246\,w_2 = 2\,913{,}90 \qquad (\text{}_{\mathrm{b}})$$

jednadžba rada na virtualnom pomaku $\delta u_{2,3}$:

$$\left(M_{1,2} + M_{2,1}\right)\delta\psi_{\{1,2\}}^{(\delta u_{\{2,3\}})} + M_{2,3}\,\delta\psi_{\{2,3\}}^{(\delta u_{\{2,3\}})} + M_{4,3}\,\delta\psi_{\{3,4\}}^{(\delta u_{\{2,3\}})} = 0$$
$$\left(M_{1,2} + M_{2,1}\right) \cdot 0 + M_{2,3} \cdot 0 + M_{4,3}\,\delta\psi_{\{3,4\}}^{(\delta u_{\{2,3\}})} = 0$$
$$\left(-\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} + \overline{M}_{4,3}^{\mathrm{c}}\right)\frac{\cancel{\delta u_{2,3}}}{2} = 0$$
$$-60\,750\,u_{\{2,3\}} = 72{,}9 \quad \Big| \quad \times (-1)$$
$$60\,750\,u_{\{2,3\}} = -72{,}9 \qquad (\text{}_{\mathrm{b}})$$

rješenje sustava jednadžbi (), () & ():

iz () neposredno: $u_{\{2,3\}} \;=\; -0{,}001\,2$ m

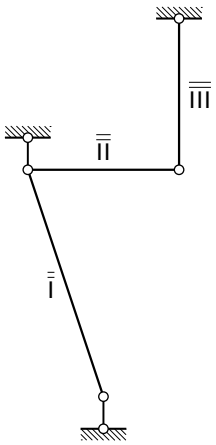
iz () & (): $\varphi_2 \;=\; -0{,}000\,090\,25$ & $w_2 \;=\; 0{,}002\,207$ m

vrijednosti momenata na krajevima štapova:

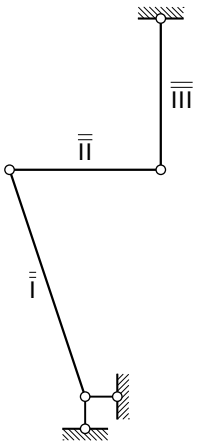
$$\begin{aligned}
M_{1,2} \;&=\; k_{\{1,2\}}\left(2\,\varphi_2 \;-\; 6\,w_2\right) \;+\; \overline{M}_{1,2} \\
&=\; 102\,458\cdot\left[2\cdot(-0{,}000\,090\,25) \;-\; 6\cdot 0{,}002\,207\right] \;+\; 1\,230{,}15 \;=\; -145{,}09\ \text{kNm} \\
M_{2,1} \;&=\; k_{\{1,2\}}\left(4\,\varphi_2 \;-\; 6\,w_2\right) \;+\; \overline{M}_{1,2} \\
&=\; 102\,458\cdot\left[4\cdot(-0{,}000\,090\,25) \;-\; 6\cdot 0{,}002\,207\right] \;+\; 1\,683{,}75 \;=\; 290{,}01\ \text{kNm} \\
M_{2,3} \;&=\; k_{\{2,3\}}\left(3\,\varphi_2 \;-\; \frac{3}{2}\,w_2\right) \;+\; \overline{M}^{c}_{2,3} \\
&=\; 81\,000\cdot\left[3\cdot(-0{,}000\,090\,25) \;-\; \frac{3}{2}\cdot 0{,}002\,207\right] \;+\; 0 \;=\; -290{,}08\ \text{kNm} \\
M_{4,3} \;&=\; -\frac{3}{2}\,k_{\{3,4\}}\,u_{\{2,3\}} \;+\; \overline{M}^{c}_{4,3} \;=\; -\frac{3}{2}\cdot 81\,000\cdot(-0{,}001\,2) \;-\; 145{,}80 \;=\; 0
\end{aligned}$$

momentni dijagram na vrhu stranice 8

c.

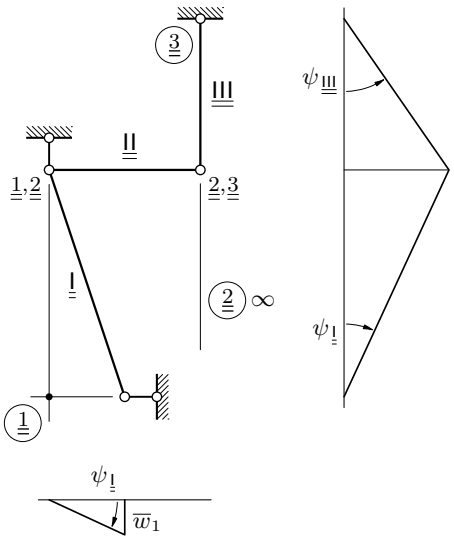


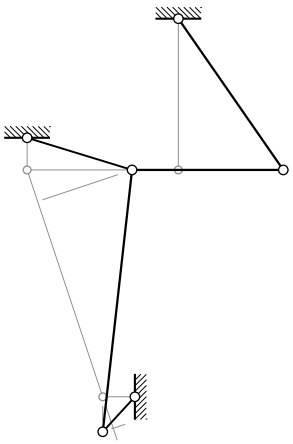
isti mehanizam kao za $\vec{u}_{\{2,3\}}$ u b.,
ali s drugim „vodećim” pomakom
— nastavite sami!

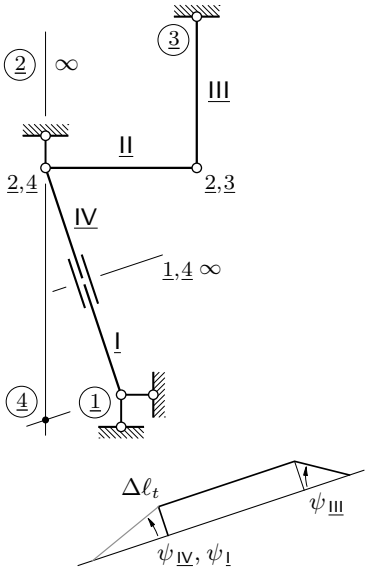


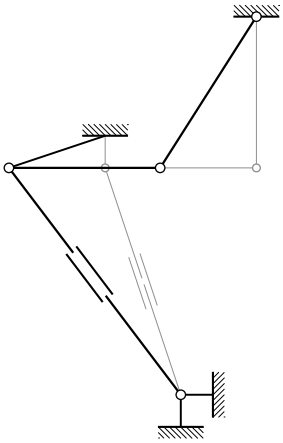
isti mehanizam kao za $\vec{u}_{\{2,3\}}$ u a.,
ali s drugim „vodećim” pomakom
— nastavite sami!

Prije no što okrenete stranicu, pokušajte sami nacrtati dijagrame projekcija pomakā i(li) planove pomakā za zadani prisilni pomak i za utjecaj promjene temperature!









Proračun provedite sami!