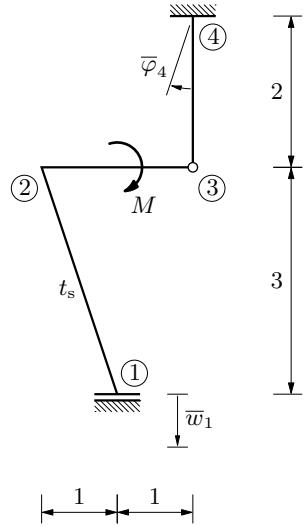


Inženjerska metoda pomakā

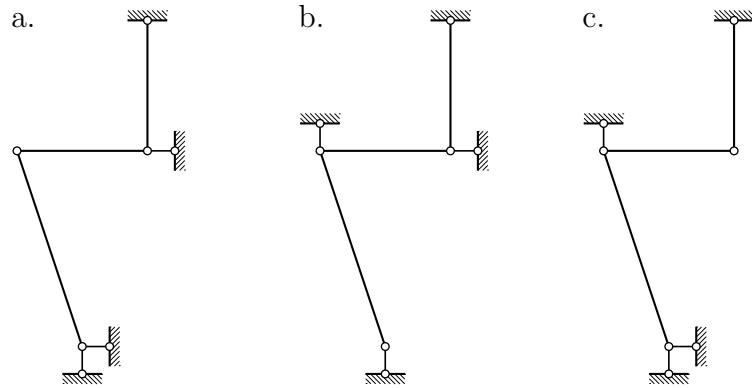
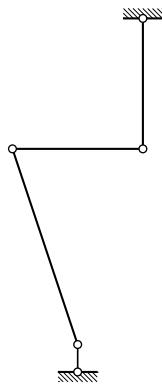
Primjer

Nacrtajte dijagram momenata savijanja!



$$\begin{aligned}
 \bar{w}_1 &= 3 \text{ mm} \\
 \bar{\varphi}_4 &= 0,0006 \\
 M &= 360 \text{ kNm} \\
 t_s &= 21^\circ \text{C} \\
 \alpha_t &= 10^{-5} \text{ K}^{-1} \\
 EI &= 162\,000 \text{ kNm}^2 \\
 (EI)_{\{2,3\}} &= (EI)_{\{3,4\}} = EI \\
 (EI)_{\{1,2\}} &= 2EI
 \end{aligned}$$

nepoznanice:



$$\begin{aligned}
 S_{\min} &= 3 \cdot 2 - 4 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \varphi_2 & \\
 u_1, u_{\{2,3\}} &
 \end{aligned}$$

$$\begin{aligned}
 \varphi_2 & \\
 w_2, u_{\{2,3\}} &
 \end{aligned}$$

$$\begin{aligned}
 \varphi_2 & \\
 u_1, w_2 &
 \end{aligned}$$

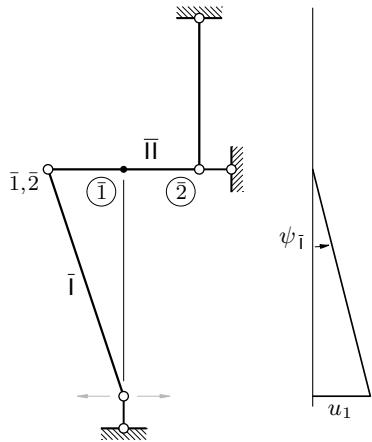
a.

$$M_{1,2} = 2k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(u_1)} - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(u_{\{2,3\}})} + \bar{M}_{1,2}$$

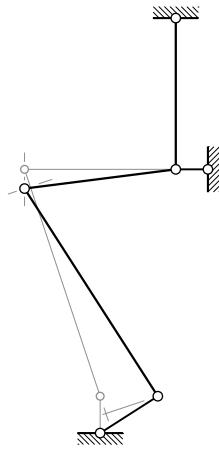
$$M_{2,1} = 4k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(u_1)} - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(u_{\{2,3\}})} + \bar{M}_{2,1}$$

$$M_{2,3} = M_{2,3}^c = 3k_{\{2,3\}}\varphi_2 - 3k_{\{2,3\}}\psi_{\{2,3\}}^{(u_1)} - 3k_{\{2,3\}}\psi_{\{2,3\}}^{(u_{\{2,3\}})} + \bar{M}_{2,3}^c$$

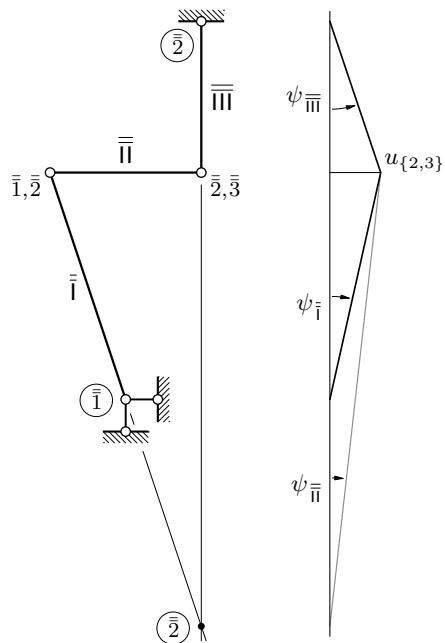
$$M_{4,3} = M_{4,3}^c = -3k_{\{3,4\}}\psi_{\{3,4\}}^{(u_1)} - 3k_{\{3,4\}}\psi_{\{3,4\}}^{(u_{\{2,3\}})} + \bar{M}_{4,3}^c$$



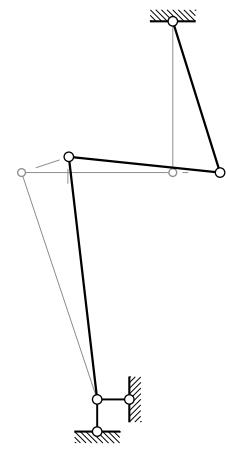
$$\begin{aligned}\psi_{\{1,2\}}^{(u_1)} &= \psi_{\bar{1}} = \frac{u_1}{3} \\ w_2^{(u_1)} &= \psi_{\bar{1}} \cdot 1 \\ \psi_{\{2,3\}}^{(u_1)} &= \psi_{\bar{II}} = \frac{w_2^{(u_1)}}{2} \\ &= \frac{\psi_{\bar{1}}}{2} = \frac{u_1}{6} \\ \psi_{\{3,4\}}^{(u_1)} &= 0\end{aligned}$$



$$w_2^{(u_1)} \quad \psi_{\bar{1}} \quad \psi_{\bar{II}}$$



$$\begin{aligned}\psi_{\{1,2\}}^{(u_{\{2,3\}})} &= \psi_{\bar{1}} = -\frac{u_{\{2,3\}}}{3} \\ \psi_{\{2,3\}}^{(u_{\{2,3\}})} &= \psi_{\bar{II}} = -\frac{u_{\{2,3\}}}{6} \\ \psi_{\{3,4\}}^{(u_{\{2,3\}})} &= \psi_{\bar{III}} = \frac{u_{\{2,3\}}}{2}\end{aligned}$$

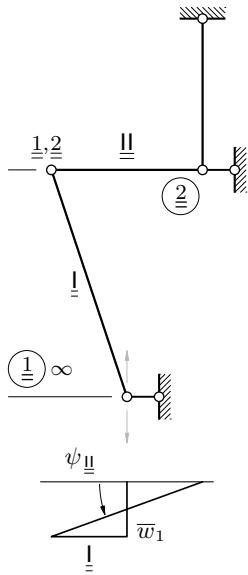


$$\begin{aligned}M_{1,2} &= 2k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}\frac{u_1}{3} - 6k_{\{1,2\}}\left(-\frac{u_{\{2,3\}}}{3}\right) + \bar{M}_{1,2} \\ &= 2k_{\{1,2\}}\varphi_2 - 2k_{\{1,2\}}u_1 + 2k_{\{1,2\}}u_{\{2,3\}} + \bar{M}_{1,2}\end{aligned}$$

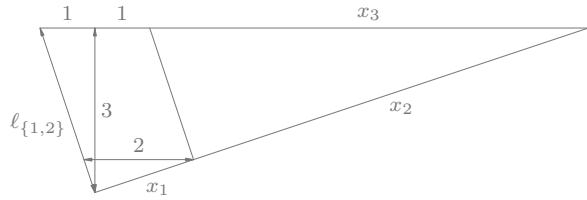
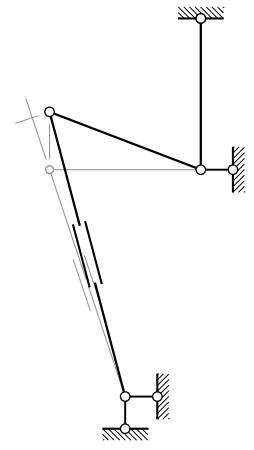
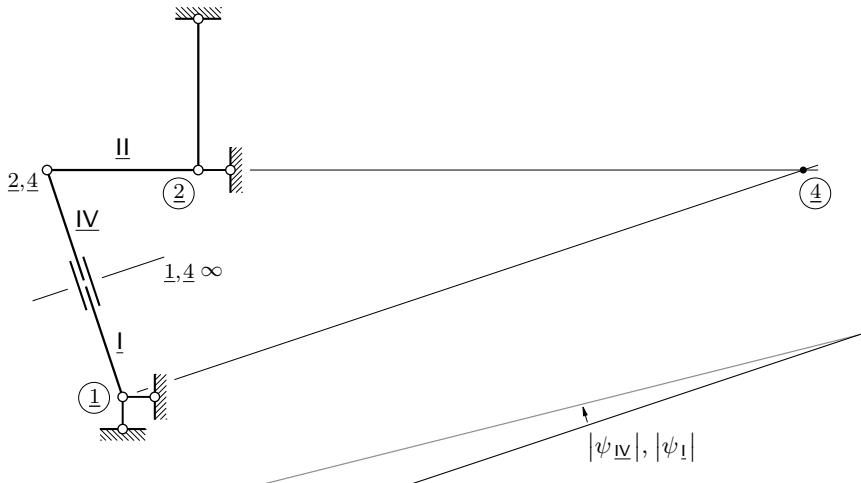
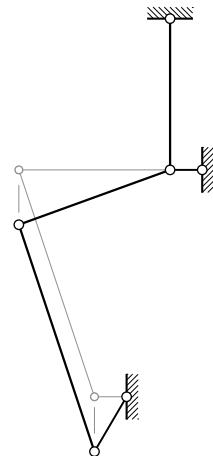
$$\begin{aligned}M_{2,1} &= 4k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}\frac{u_1}{3} - 6k_{\{1,2\}}\left(-\frac{u_{\{2,3\}}}{3}\right) + \bar{M}_{2,1} \\ &= 4k_{\{1,2\}}\varphi_2 - 2k_{\{1,2\}}u_1 + 2k_{\{1,2\}}u_{\{2,3\}} + \bar{M}_{2,1}\end{aligned}$$

$$\begin{aligned}M_{2,3} &= M_{2,3}^c = 3k_{\{2,3\}}\varphi_2 - 3k_{\{2,3\}}\frac{u_1}{6} - 3k_{\{2,3\}}\left(-\frac{u_{\{2,3\}}}{6}\right) + \bar{M}_{2,3}^c \\ &= 3k_{\{2,3\}}\varphi_2 - \frac{1}{2}k_{\{2,3\}}u_1 + \frac{1}{2}k_{\{2,3\}}u_{\{2,3\}} + \bar{M}_{2,3}^c\end{aligned}$$

$$M_{4,3} = M_{4,3}^c = -3k_{\{3,4\}} \cdot 0 - 3k_{\{3,4\}}\frac{u_{\{2,3\}}}{2} + \bar{M}_{4,3}^c = -\frac{3}{2}k_{\{3,4\}}u_{\{2,3\}} + \bar{M}_{4,3}^c$$



$$\begin{aligned}\psi_{\{1,2\}}^{(\bar{w}_1)} &= \psi_{\underline{1}} = 0 \\ \psi_{\{2,3\}}^{(\bar{w}_1)} &= \psi_{\underline{1}} = \frac{\bar{w}_1}{2} \\ &= \frac{0,003}{2} = 0,0015 \\ \psi_{\{3,4\}}^{(\bar{w}_1)} &= 0\end{aligned}$$



$$\ell_{\{1,2\}} = \sqrt{1^2 + 9^2} = \sqrt{10} = 3,16 \text{ m}$$

$$\Delta\ell_t = \alpha_t t_s \ell_{\{1,2\}} = 10^{-5} \cdot 21 \cdot \sqrt{10} = 0,000\,664 \text{ m}$$

$$\frac{x_1}{2} = \frac{3}{\ell_{\{1,2\}}} \quad \Rightarrow \quad x_1 = \frac{6}{\ell_{\{1,2\}}} = \frac{6}{\sqrt{10}} = 1,90 \text{ m}$$

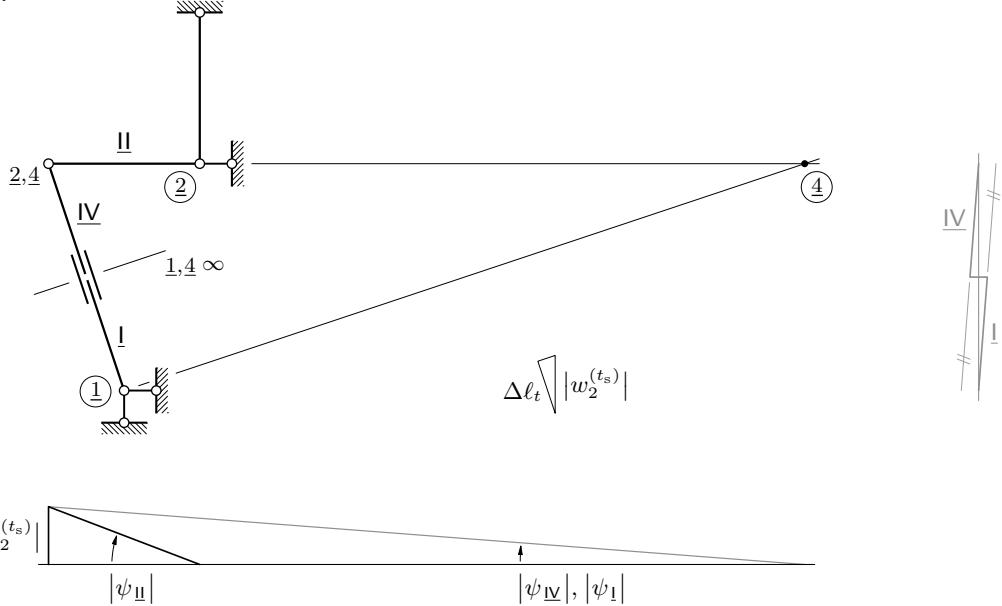
$$\frac{x_1 + x_2}{\ell_{\{1,2\}}} = \frac{3}{1} \Rightarrow x_1 + x_2 = 3 \ell_{\{1,2\}} = 9,48 \text{ m}$$

$$\psi_{\{2,3\}}^{(t_s)} = \psi_{\underline{\underline{1}}} = -\frac{\Delta\ell_t}{x_1} = -\alpha_t t_s \ell_{\{1,2\}} \frac{\ell_{\{1,2\}}}{6} = -10^{-5} \cdot 21 \cdot \frac{(\sqrt{10})^2}{6} = -0,000350$$

$$\psi_{\{1,2\}}^{(t_s)} = \psi_{\underline{1}} = -\frac{\Delta\ell_t}{x_1 + x_2} = -\frac{\alpha_t t_s \ell_{\{1,2\}}}{3 \ell_{\{1,2\}}} = -0,0000700$$

$$\psi_{\{3,4\}}^{(t_s)} = 0$$

ili:



$$\frac{|w_2^{(t_s)}|}{\Delta\ell_t} = \frac{\ell_{\{1,2\}}}{3} \Rightarrow |w_2^{(t_s)}| = \Delta\ell_t \frac{\ell_{\{1,2\}}}{3} = -10^{-5} \cdot 21 \cdot \frac{(\sqrt{10})^2}{3} = 0,000700 \text{ m}$$

(za objašnjenje trokuta sa stranicama $\Delta\ell_t$ i $|w_2^{(t_s)}|$ pročitajte tekst na vrhu stranice 4 u datoteci <http://master.grad.hr/nastava/gs/gs2/isp/2-24-pk-1.pdf>)

$$\psi_{\{2,3\}}^{(t_s)} = \psi_{\underline{\underline{1}}} = -\frac{|w_2^{(t_s)}|}{2} = -0,000350$$

$$\frac{2 + x_3}{\ell_{\{1,2\}}} = \frac{\ell_{\{1,2\}}}{1} \Rightarrow 2 + x_3 = \ell_{\{1,2\}}^2 = 10 \text{ m} \quad (\text{prema skici na prethodnoj stranici})$$

$$\psi_{\{1,2\}}^{(t_s)} = \psi_{\underline{1}} = -\frac{|w_2^{(t_s)}|}{2 + x_3} = -0,0000700$$

$$k_{\{1,2\}} = \frac{2EI}{\ell_{\{1,2\}}} = \frac{2 \cdot 162\,000}{\sqrt{10}} = 102\,458 \text{ kNm}$$

$$k_{\{2,3\}} = \frac{EI}{\ell_{\{2,3\}}} = \frac{162\,000}{2} = 81\,000 \text{ kNm}, \quad k_{\{3,4\}} = \frac{EI}{\ell_{\{3,4\}}} = 81\,000 \text{ kNm}$$

$$\begin{aligned}
\bar{M}_{1,2} &= \bar{M}_{1,2}^{(\bar{w}_1)} + \bar{M}_{1,2}^{(t_s)} = -6 k_{\{1,2\}} \psi_{\{1,2\}}^{(\bar{w}_1)} - 6 k_{\{1,2\}} \psi_{\{1,2\}}^{(t_s)} \\
&= -6 k_{\{1,2\}} \cdot 0 - 6 k_{\{1,2\}} \psi_{\{1,2\}}^{(t_s)} = -6 \cdot 102458 \cdot (-0,00007) = 43,03 \text{ kNm} \\
\bar{M}_{2,1} &= \bar{M}_{2,1}^{(\bar{w}_1)} + \bar{M}_{2,1}^{(t_s)} = -6 k_{\{1,2\}} \psi_{\{1,2\}}^{(\bar{w}_1)} - 6 k_{\{1,2\}} \psi_{\{1,2\}}^{(t_s)} = \bar{M}_{1,2} = 43,03 \text{ kNm} \\
\bar{M}_{2,3}^c &= {}^c \bar{M}_{2,3}^{(\bar{w}_1)} + {}^c \bar{M}_{2,3}^{(t_s)} + {}^c \bar{M}_{2,3}^{(M)} = {}^c \bar{M}_{2,3}^{(\bar{w}_1)} + {}^c \bar{M}_{2,3}^{(t_s)} + \left(\bar{M}_{2,3}^{(M)} - \frac{1}{2} \bar{M}_{3,2}^{(M)} \right) \\
&= -3 k_{\{2,3\}} \psi_{\{2,3\}}^{(\bar{w}_1)} - 3 k_{\{2,3\}} \psi_{\{2,3\}}^{(t_s)} + \left(\frac{M}{4} - \frac{1}{2} \frac{M}{4} \right) \\
&= -3 \cdot 81000 \cdot 0,0015 - 3 \cdot 81000 \cdot (-0,00035) + \frac{-360}{8} = -324,45 \text{ kNm} \\
\bar{M}_{4,3}^c &= {}^c \bar{M}_{4,3}^{(\bar{\varphi}_4)} = -3 k_{\{3,4\}} \bar{\varphi}_4 = -3 \cdot 81000 \cdot (-0,0006) = -145,80 \text{ kNm}
\end{aligned}$$

jednadžba ravnoteže momenata u čvoru 2:

$$\begin{aligned}
-M_{2,1} - M_{2,3} &= 0 \quad \Rightarrow \quad M_{2,1} + M_{2,3} = 0 \\
[4 k_{\{1,2\}} \varphi_2 - 2 k_{\{1,2\}} u_1 + 2 k_{\{1,2\}} u_{\{2,3\}} + \bar{M}_{2,1}] \\
+ \left[3 k_{\{2,3\}} \varphi_2 - \frac{1}{2} k_{\{2,3\}} u_1 + \frac{1}{2} k_{\{2,3\}} u_{\{2,3\}} + \bar{M}_{2,3}^c \right] &= 0 \\
(4 k_{\{1,2\}} + 3 k_{\{2,3\}}) \varphi_2 + \left(-2 k_{\{1,2\}} - \frac{1}{2} k_{\{2,3\}} \right) u_1 \\
+ \left(2 k_{\{1,2\}} + \frac{1}{2} k_{\{2,3\}} \right) u_{\{2,3\}} &= -\bar{M}_{2,1} - \bar{M}_{2,3}^c \\
652832 \cdot \varphi_2 - 245416 \cdot u_1 + 245416 \cdot u_{\{2,3\}} &= 281,42 \tag{↔↔↔}
\end{aligned}$$

jednadžba rada na virtualnom pomaku δu_1 (projekcije pomaka za u_1 uz dodani δ):

$$\begin{aligned}
(M_{1,2} + M_{2,1}) \delta \psi_{\{1,2\}}^{(\delta u_1)} + M_{2,3} \delta \psi_{\{2,3\}}^{(\delta u_1)} + M_{4,3} \delta \psi_{\{3,4\}}^{(\delta u_1)} + (-M) \delta \psi_{\{2,3\}}^{(\delta u_1)} &= 0 \\
(6 k_{\{1,2\}} \varphi_2 - 4 k_{\{1,2\}} u_1 + 4 k_{\{1,2\}} u_{\{2,3\}} + \bar{M}_{1,2} + \bar{M}_{2,1}) \frac{\delta u_1}{3} \\
+ \left(3 k_{\{2,3\}} \varphi_2 - \frac{1}{2} k_{\{2,3\}} u_1 + \frac{1}{2} k_{\{2,3\}} u_{\{2,3\}} + \bar{M}_{2,3}^c \right) \frac{\delta u_1}{6} + 0 - M \frac{\delta u_1}{6} &= 0 \\
\left(2 k_{\{1,2\}} + \frac{1}{2} k_{\{2,3\}} \right) \varphi_2 - \left(\frac{4}{3} k_{\{1,2\}} + \frac{1}{12} k_{\{2,3\}} \right) u_1 \\
+ \left(\frac{4}{3} k_{\{1,2\}} + \frac{1}{12} k_{\{2,3\}} \right) u_{\{2,3\}} &= -\frac{1}{3} (\bar{M}_{1,2} + \bar{M}_{2,1}) - \frac{1}{6} (\bar{M}_{2,3}^c - M) \\
245416 \varphi_2 - 143361 u_1 + 143361 u_{\{2,3\}} &= 85,39 \quad | \quad \times (-1) \\
-245416 \varphi_2 + 143361 u_1 - 143361 u_{\{2,3\}} &= -85,39 \tag{→→→}
\end{aligned}$$

jednadžba rada na virtualnom pomaku $\delta u_{2,3}$ (projekcije pomaka za $u_{2,3}$ uz dodani δ):

$$\begin{aligned}
 & (M_{1,2} + M_{2,1}) \delta\psi_{\{1,2\}}^{(\delta u_{\{2,3\}})} + M_{2,3} \delta\psi_{\{2,3\}}^{(\delta u_{\{2,3\}})} + M_{4,3} \delta\psi_{\{3,4\}}^{(\delta u_{\{2,3\}})} + (-M) \delta\psi_{\{2,3\}}^{(\delta u_{\{2,3\}})} = 0 \\
 & (6 k_{\{1,2\}} \varphi_2 - 4 k_{\{1,2\}} u_1 + 4 k_{\{1,2\}} u_{2,3} + \bar{M}_{1,2} + \bar{M}_{2,1}) \left(-\frac{\delta u_{2,3}}{3} \right) \\
 & + \left(3 k_{\{2,3\}} \varphi_2 - \frac{1}{2} k_{\{2,3\}} u_1 + \frac{1}{2} k_{\{2,3\}} u_{2,3} + \bar{M}_{2,3}^c \right) \left(-\frac{\delta u_{2,3}}{6} \right) \\
 & + \left(-\frac{3}{2} k_{\{3,4\}} u_{2,3} + \bar{M}_{4,3}^c \right) \frac{\delta u_{2,3}}{2} - M \left(-\frac{\delta u_{2,3}}{6} \right) = 0 \\
 & - \left(2 k_{\{1,2\}} + \frac{1}{2} k_{\{2,3\}} \right) \varphi_2 + \left(\frac{4}{3} k_{\{1,2\}} + \frac{1}{12} k_{\{2,3\}} \right) u_1 \\
 & - \left(\frac{4}{3} k_{\{1,2\}} + \frac{1}{12} k_{\{2,3\}} + \frac{3}{4} k_{\{3,4\}} \right) u_{2,3} \\
 & = \frac{1}{3} (\bar{M}_{1,2} + \bar{M}_{2,1}) + \frac{1}{6} (\bar{M}_{2,3}^c - M) - \frac{1}{2} \bar{M}_{4,3}^c \\
 & -245\,416 \varphi_2 + 143\,361 u_1 - 204\,111 u_{2,3} = -12,49 \quad | \quad \times (-1) \\
 & 245\,416 \varphi_2 - 143\,361 u_1 + 204\,111 u_{2,3} = 12,49 \quad (\text{---})
 \end{aligned}$$

rješenje sustava jednadžbi (---), (---) i (---):

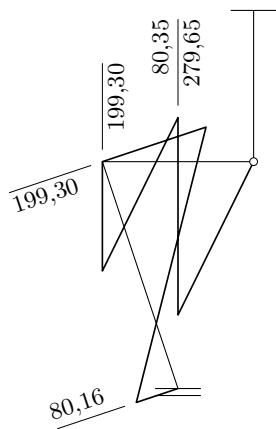
$$\varphi_2 = 0,000\,581\,2, \quad u_1 = -0,000\,8 \text{ m} \quad \& \quad u_{\{2,3\}} = -0,001\,2 \text{ m}$$

vrijednosti momenata na krajevima štapova:

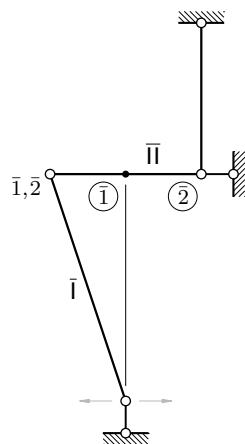
$$\begin{aligned}
 M_{1,2} &= k_{\{1,2\}} (2 \varphi_2 - 2 u_1 + 2 u_{\{2,3\}}) + \bar{M}_{1,2} \\
 &= 102\,458 \cdot [2 \cdot 0,000\,581\,2 - 2 \cdot (-0,000\,8) + 2 \cdot (-0,001\,2)] + 43,03 \\
 &= 80,16 \text{ kNm} \\
 M_{2,1} &= k_{\{1,2\}} (4 \varphi_2 - 2 u_1 + 2 u_{\{2,3\}}) + \bar{M}_{2,1} \\
 &= 102\,458 \cdot [4 \cdot 0,000\,581\,2 - 2 \cdot (-0,000\,8) + 2 \cdot (-0,001\,2)] + 43,03 \\
 &= 199,26 \text{ kNm} \\
 M_{2,3} &= k_{\{2,3\}} \left(3 \varphi_2 - \frac{1}{2} u_1 + \frac{1}{2} u_{\{2,3\}} \right) + \bar{M}_{2,3}^c \\
 &= 81\,000 \cdot \left[3 \cdot 0,000\,581\,2 - \frac{1}{2} \cdot (-0,000\,8) + \frac{1}{2} \cdot (-0,001\,2) \right] - 324,45 \\
 &= -199,40 \text{ kNm} \\
 M_{4,3} &= -\frac{3}{2} k_{\{3,4\}} u_{\{2,3\}} + \bar{M}_{4,3}^c = -\frac{3}{2} \cdot 81\,000 \cdot (-0,001\,2) - 145,80 = 0
 \end{aligned}$$

(„neravnoteža“ momenata u čvoru 2 posljedica je pogrešaka zaokruživanja)

momentni dijagram:



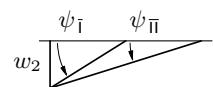
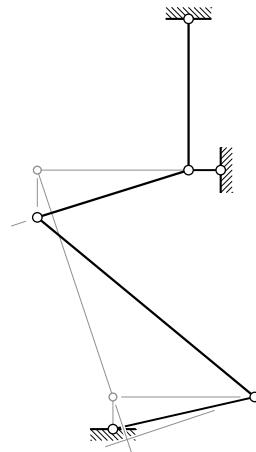
b.



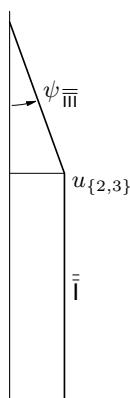
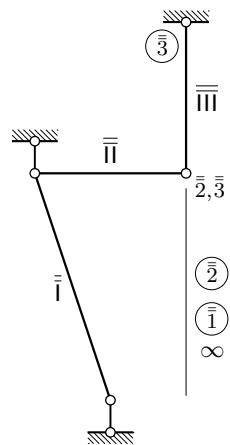
$$\psi_{\{1,2\}}^{(w_2)} = \psi_I = \frac{w_2}{1}$$

$$\psi_{\{2,3\}}^{(w_2)} = \psi_{II} = \frac{w_2}{2}$$

$$\psi_{\{3,4\}}^{(w_2)} = 0$$



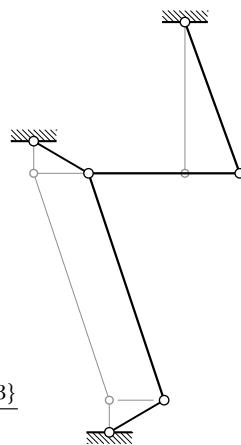
(isti mehanizam kao za \vec{u}_1 u a., ali je „vodeći” pomak sada \vec{w}_2)



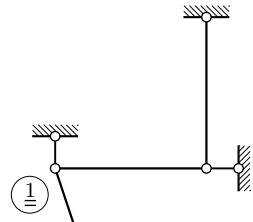
$$\psi_{\{1,2\}}^{(u_{\{2,3\}})} = 0$$

$$\psi_{\{2,3\}}^{(u_{\{2,3\}})} = 0$$

$$\psi_{\{3,4\}}^{(u_{\{2,3\}})} = \psi_{III} = \frac{u_{\{2,3\}}}{2}$$

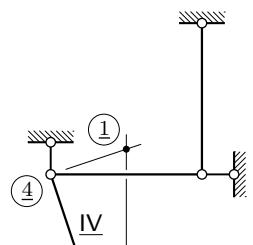
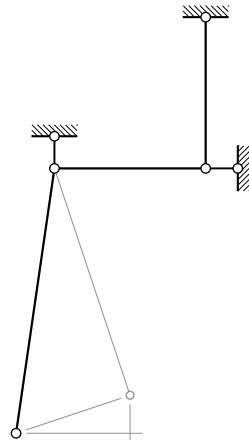


$$\begin{aligned}
M_{1,2} &= 2k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(w_2)} - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(u_{\{2,3\}})} + \bar{M}_{1,2} \\
&= 2k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}w_2 + \bar{M}_{1,2} \\
M_{2,1} &= 4k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(w_2)} - 6k_{\{1,2\}}\psi_{\{1,2\}}^{(u_{\{2,3\}})} + \bar{M}_{2,1} \\
&= 4k_{\{1,2\}}\varphi_2 - 6k_{\{1,2\}}w_2 + \bar{M}_{2,1} \\
M_{2,3} &= M_{2,3}^c = 3k_{\{2,3\}}\varphi_2 - 3k_{\{2,3\}}\psi_{\{2,3\}}^{(w_2)} - 3k_{\{2,3\}}\psi_{\{2,3\}}^{(u_{\{2,3\}})} + \bar{M}_{2,3}^c \\
&= 3k_{\{2,3\}}\varphi_2 - \frac{3}{2}k_{\{2,3\}}w_2 + \bar{M}_{2,3}^c \\
M_{4,3} &= M_{4,3}^c = -3k_{\{3,4\}}\psi_{\{3,4\}}^{(w_2)} - 3k_{\{3,4\}}\psi_{\{3,4\}}^{(u_{\{2,3\}})} + \bar{M}_{4,3}^c \\
&= -\frac{3}{2}k_{\{3,4\}}u_{\{2,3\}} + \bar{M}_{4,3}^c
\end{aligned}$$



$$\begin{aligned}
\psi_{\{1,2\}}^{(\bar{w}_1)} &= \psi_{\underline{1}} = -\frac{\bar{w}_1}{1} \\
&= -0,003
\end{aligned}$$

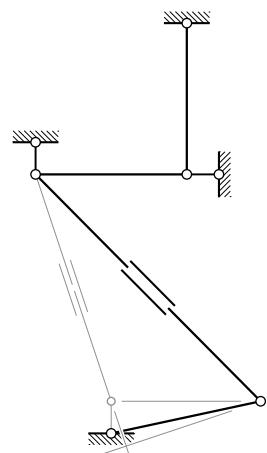
$$\frac{\psi_{\underline{1}}}{\bar{w}_1}$$



$$\begin{aligned}
\frac{x}{1} &= \frac{\ell_{\{1,2\}}}{3} \Rightarrow x = \frac{\ell_{\{1,2\}}}{3} \\
\psi_{\{1,2\}}^{(t_s)} &= \psi_{\underline{1}} = \frac{\Delta\ell_t}{x} \\
&= \frac{3\Delta\ell_t}{\ell_{\{1,2\}}} = 0,000\,630
\end{aligned}$$

$$\begin{array}{l}
\psi_{\underline{1}}, \psi_{\underline{IV}} \\
\Delta\ell_t \\
x
\end{array}$$

$$\psi_{\{2,3\}}^{(t_s)} = \psi_{\{3,4\}}^{(t_s)} = 0$$



$$\begin{aligned}\bar{M}_{1,2} &= \bar{M}_{1,2}^{(\bar{w}_1)} + \bar{M}_{1,2}^{(t_s)} = -6 k_{\{1,2\}} \psi_{\{1,2\}}^{(\bar{w}_1)} - 6 k_{\{1,2\}} \psi_{\{1,2\}}^{(t_s)} \\ &= -6 \cdot 102458 \cdot (-0,003) - 6 \cdot 102458 \cdot 0,00063 = 1456,95 \text{ kNm}\end{aligned}$$

$$\bar{M}_{2,1} = \bar{M}_{1,2} = 1456,95 \text{ kNm}$$

$$\begin{aligned}\bar{M}_{2,3}^c &= {}^c\bar{M}_{2,3}^{(\bar{w}_1)} + {}^c\bar{M}_{2,3}^{(t_s)} + {}^c\bar{M}_{2,3}^{(M)} \\ &= -3 k_{\{2,3\}} \psi_{\{2,3\}}^{(\bar{w}_1)} - 3 k_{\{2,3\}} \psi_{\{2,3\}}^{(t_s)} + \frac{M}{8} = \frac{-360}{8} = -45 \text{ kNm} \\ \bar{M}_{4,3}^c &= {}^c\bar{M}_{4,3}^{(\bar{\varphi}_4)} = -3 k_{\{3,4\}} \bar{\varphi}_4 = -3 \cdot 81000 \cdot (-0,0006) = -145,80 \text{ kNm}\end{aligned}$$

jednadžba ravnoteže momenata u čvoru 2:

$$\begin{aligned}-M_{2,1} - M_{2,3} &= 0 \quad \Rightarrow \quad M_{2,1} + M_{2,3} = 0 \\ [4 k_{\{1,2\}} \varphi_2 - 6 k_{\{1,2\}} w_2 + \bar{M}_{2,1}] + \left[3 k_{\{2,3\}} \varphi_2 - \frac{3}{2} k_{\{2,3\}} w_2 + \bar{M}_{2,3}^c\right] &= 0 \\ (4 k_{\{1,2\}} + 3 k_{\{2,3\}}) \varphi_2 + \left(-6 k_{\{1,2\}} - \frac{3}{2} k_{\{2,3\}}\right) w_2 &= -\bar{M}_{2,1} - \bar{M}_{2,3}^c \\ 652832 \cdot \varphi_2 - 736248 \cdot w_2 &= -1411,95 \quad (\text{--- b})\end{aligned}$$

jednadžba rada na virtualnom pomaku δw_2 :

$$\begin{aligned}(M_{1,2} + M_{2,1}) \delta \psi_{\{1,2\}}^{(\delta w_2)} + M_{2,3} \delta \psi_{\{2,3\}}^{(\delta w_2)} + M_{4,3} \delta \psi_{\{3,4\}}^{(\delta w_2)} + (-M) \delta \psi_{\{2,3\}}^{(\delta w_2)} &= 0 \\ (6 k_{\{1,2\}} \varphi_2 - 12 k_{\{1,2\}} w_2 + \bar{M}_{1,2} + \bar{M}_{2,1}) \delta w_2 &+ \\ + \left(3 k_{\{2,3\}} \varphi_2 - \frac{3}{2} k_{\{2,3\}} w_2 + \bar{M}_{2,3}^c\right) \frac{\delta w_2}{2} - M \frac{\delta w_2}{2} &= 0 \\ \left(6 k_{\{1,2\}} + \frac{3}{2} k_{\{2,3\}}\right) \varphi_2 - \left(12 k_{\{1,2\}} + \frac{3}{4} k_{\{2,3\}}\right) w_2 &= -\bar{M}_{1,2} - \bar{M}_{2,1} - \frac{1}{2} (\bar{M}_{2,3}^c - M) \\ 736248 \varphi_2 - 1290246 w_2 &= -2711,4 \quad | \quad \times (-1) \\ -736248 \varphi_2 + 1290246 w_2 &= 2711,4 \quad (\text{--- b})\end{aligned}$$

jednadžba rada na virtualnom pomaku $\delta u_{2,3}$:

$$\begin{aligned}(M_{1,2} + M_{2,1}) \delta \psi_{\{1,2\}}^{(\delta u_{2,3})} + M_{2,3} \delta \psi_{\{2,3\}}^{(\delta u_{2,3})} + M_{4,3} \delta \psi_{\{3,4\}}^{(\delta u_{2,3})} + (-M) \delta \psi_{\{2,3\}}^{(\delta u_{2,3})} &= 0 \\ \left(-\frac{3}{2} k_{\{3,4\}} u_{\{2,3\}} + \bar{M}_{4,3}^c\right) \frac{\delta u_{2,3}}{2} &= 0 \\ -60750 u_{\{2,3\}} &= 72,9 \quad | \quad \times (-1) \\ 60750 u_{\{2,3\}} &= -72,9 \quad (\text{--- b})\end{aligned}$$

rješenje sustava jednadžbi (Δu_b), ($\Delta \varphi_b$) i (Δw_b):

$$\text{iz } (\Delta u_b): \quad u_{\{2,3\}} = -0,0012 \text{ m}$$

$$\text{iz } (\Delta \varphi_b) \text{ i } (\Delta w_b): \quad \varphi_2 = 0,0005812 \text{ i } w_2 = 0,002433 \text{ m}$$

vrijednosti momenata na krajevima štapova:

$$\begin{aligned} M_{1,2} &= k_{\{1,2\}} (2\varphi_2 - 6w_2) + \bar{M}_{1,2} \\ &= 102458 \cdot (2 \cdot 0,0005812 - 6 \cdot 0,002433) + 1456,95 = 80,37 \text{ kNm} \end{aligned}$$

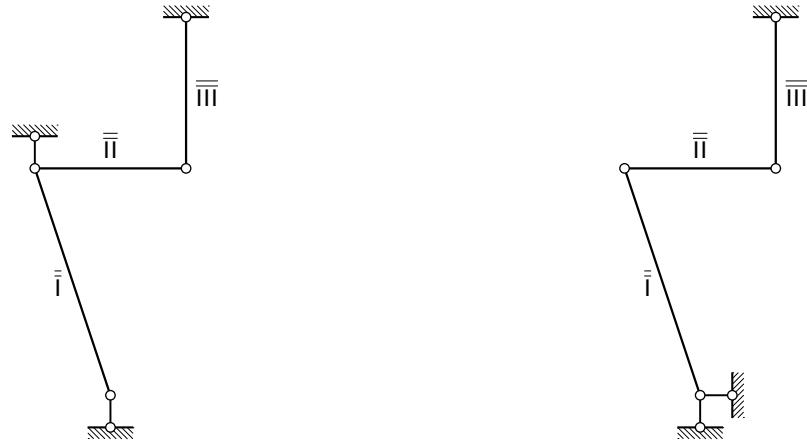
$$\begin{aligned} M_{2,1} &= k_{\{1,2\}} (4\varphi_2 - 6w_2) + \bar{M}_{1,2} \\ &= 102458 \cdot (4 \cdot 0,0005812 - 6 \cdot 0,002433) + 1456,95 = 199,46 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_{2,3} &= k_{\{2,3\}} \left(3\varphi_2 - \frac{3}{2}u_1 \right) + \bar{M}_{2,3}^c \\ &= 81000 \cdot \left(3 \cdot 0,0005812 - \frac{3}{2} \cdot 0,002433 \right) - 45 = -199,38 \text{ kNm} \end{aligned}$$

$$M_{4,3} = -\frac{3}{2} k_{\{3,4\}} u_{\{2,3\}} + \bar{M}_{4,3}^c = -\frac{3}{2} \cdot 81000 \cdot (-0,0012) - 145,80 = 0$$

momentni dijagram kao na stranici 7 (s neznatno drugačijim vrijednostima)

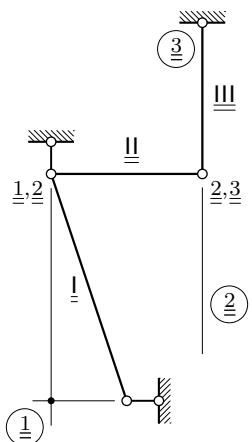
c.



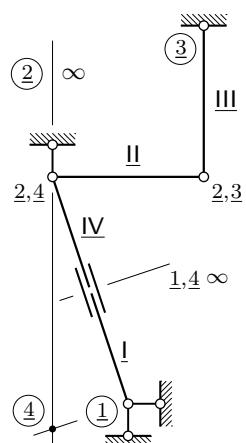
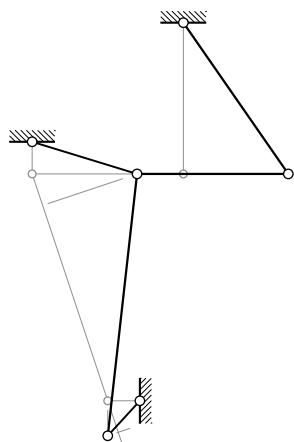
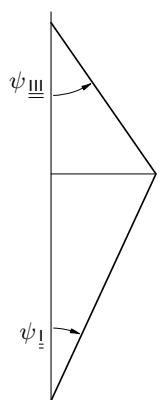
isti mehanizam kao za $\vec{u}_{\{2,3\}}$ u b.,
ali s drugim „vodećim” pomakom
— nastavite sami!

isti mehanizam kao za $\vec{u}_{\{2,3\}}$ u a.,
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— nastavite sami!

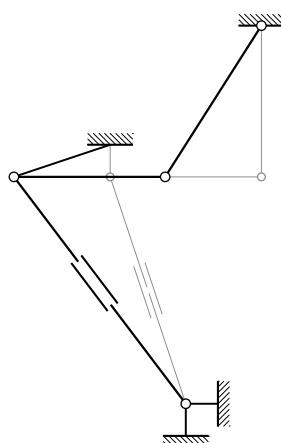
Prije no što okrenete stranicu, pokušajte sami nacrtati dijagrame projekcija pomakā i(l) planove pomakā za zadani prisilni pomak i za utjecaj promjene temperature!



$$\psi_{\frac{1}{2}} \quad \bar{w}_1$$



$$\Delta\ell_t \quad \psi_{\text{IV}}, \psi_{\text{I}}$$



Proračun provedite sam!