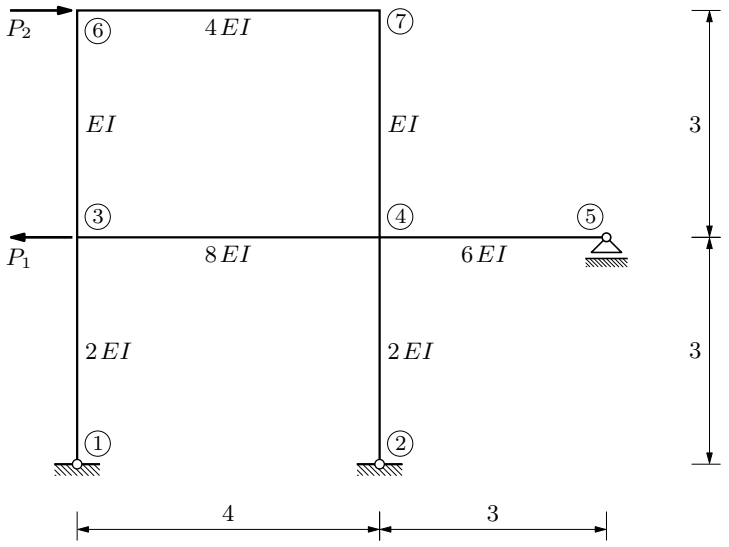
Postupak Wernera & Csonke

Primjer

Nacrtajte dijagram momenata savijanja!



$P_1 = 50$ kN

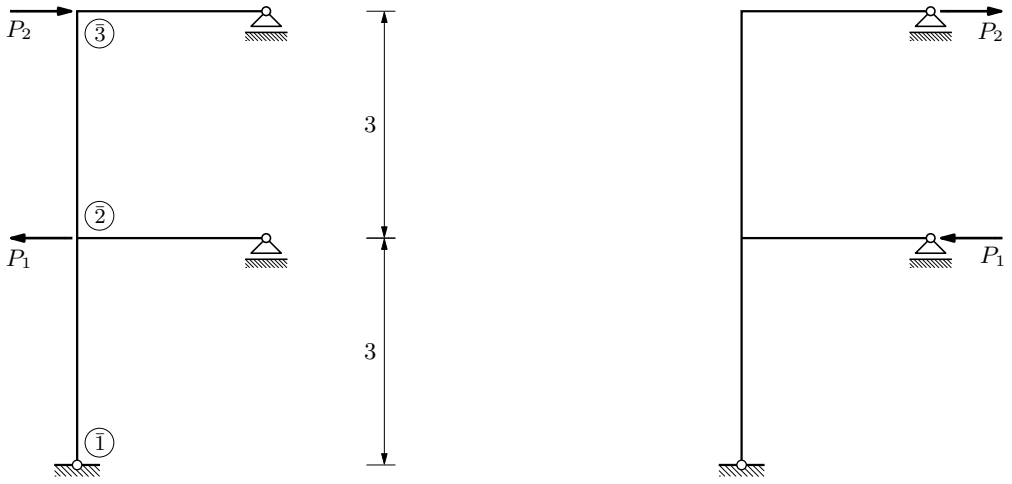
$P_2 = 100$ kN

$$k_{\{1,3\}} = k_{\{2,4\}} = \frac{2EI}{3} \qquad k_{\{3,4\}} = \frac{8EI}{4} = 2EI$$

$$k_{\{3,6\}} = k_{\{4,7\}} = \frac{EI}{3} \qquad k_{\{4,5\}} = \frac{6EI}{3} = 2EI$$

$$k_{\{6,7\}} = \frac{4EI}{4} = EI$$

zamjenjujući poluokvir:



koeficijenti krutosti elemenata poluokvira:

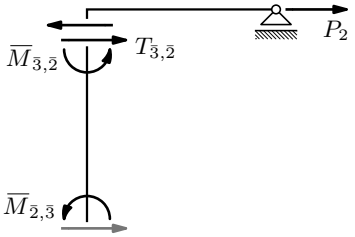
$$k_{\{\bar{1},\bar{2}\}} = k_{\{1,3\}} + k_{\{2,4\}} = \frac{2EI}{3} + \frac{2EI}{3} = \frac{4EI}{3}$$

$$k_{\{\bar{2},\bar{3}\}} \;=\; k_{\{3,6\}} \,+\, k_{\{4,7\}} \;=\; \frac{EI}{3} \,+\, \frac{EI}{3} \;=\; \frac{2\,EI}{3}$$

$$k_{\{\bar{2},\mathrm{g}\}} \;=\; 4\,k_{\{3,4\}} \,+\, k_{\{4,5\}} \;=\; 4\cdot 2\,EI \,+\, 2\,EI \;=\; 10\,EI$$

$$k_{\{\bar{3},\mathrm{g}\}} \;=\; 4\,k_{\{6,7\}} \;=\; 4\cdot EI$$

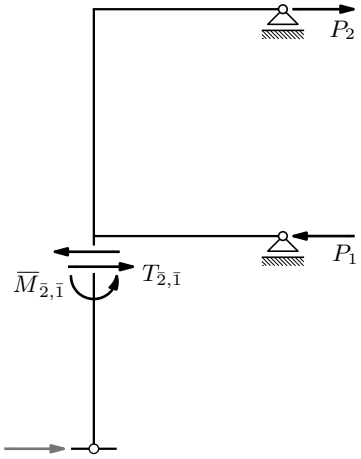
vrijednosti momenata upetosti za postupak Wernera & Csonke:



$$-T_{\bar{3},\bar{2}} \,+\, P_2 \;=\; 0$$

$$T_{\bar{3},\bar{2}} \;=\; P_2 \;=\; 100\ \mathrm{kN}$$

$$\overline{M}_{\bar{3},\bar{2}} \;=\; \overline{M}_{\bar{2},\bar{3}} \;=\; \frac{1}{2}\cdot T_{\bar{3},\bar{2}}\cdot 3 \;=\; 150\ \mathrm{kNm}$$



$$-T_{\bar{2},\bar{1}} \,+\, P_2 \,-\, P_1 \;=\; 0$$

$$T_{\bar{2},\bar{1}} \;=\; P_2 \,-\, P_1 \;=\; 100 \,-\, 50 \;=\; 50\ \mathrm{kN}$$

$$\overline{M}_{\bar{2},\bar{1}} \;=\; T_{\bar{2},\bar{1}}\cdot 3 \;=\; 150\ \mathrm{kNm}$$

razdjelni koeficijenti za postupak Wernera & Csonke:

$$k_{\bar{2}} \;=\; 3\,k_{\{\bar{2},\mathrm{g}\}} \,+\, k_{\{\bar{2},\bar{3}\}} \;=\; 3\cdot 10\,EI \,+\, \frac{2}{3}\,EI \;=\; \frac{92}{3}\,EI$$

$$\mu_{\bar{2},\mathrm{g}} \;=\; \frac{3\,k_{\{\bar{2},\mathrm{g}\}}}{k_{\bar{2}}} \;=\; \frac{30\,EI}{\frac{92}{3}\,EI} \;=\; \frac{45}{46} \;=\; 0{,}98$$

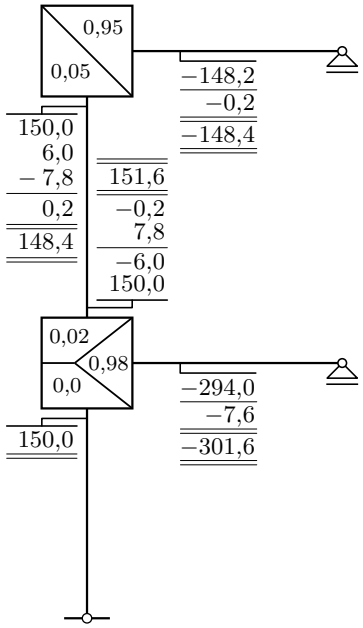
$$\mu_{\bar{2},\bar{3}} \;=\; \frac{k_{\{\bar{2},\bar{3}\}}}{k_{\bar{2}}} \;=\; \frac{\frac{2\,EI}{3}}{\frac{92}{3}\,EI} \;=\; \frac{1}{46} \;=\; 0{,}02$$

$$k_{\bar{3}} \;=\; 3\,k_{\{\bar{3},\mathrm{g}\}} \,+\, k_{\{\bar{2},\bar{3}\}} \;=\; 3\cdot 4\,EI \,+\, \frac{2}{3}\,EI \;=\; \frac{38}{3}\,EI$$

$$\mu_{\bar{3},\mathrm{g}} \;=\; \frac{3\,k_{\{\bar{3},\mathrm{g}\}}}{k_{\bar{3}}} \;=\; \frac{12\,EI}{\frac{38}{3}\,EI} \;=\; \frac{18}{19} \;=\; 0{,}95$$

$$\mu_{\bar{3},\bar{2}} \;=\; \frac{k_{\{\bar{2},\bar{3}\}}}{k_{\bar{3}}} \;=\; \frac{\frac{2\,EI}{3}}{\frac{38}{3}\,EI} \;=\; \frac{1}{19} \;=\; 0{,}05$$

iteracija (relaksacija):



$$\textcircled{2}\quad 150{,}0 + 150{,}0 = 300{,}0$$
$$-300{,}0 \cdot 0{,}98 = -294{,}0$$
$$\cdot\, 0{,}02 = -6{,}0$$

$$\textcircled{3}\quad 150{,}0 + 6{,}0 = 156{,}0$$
$$-156 \cdot 0{,}95 = -148{,}2$$
$$\cdot\, 0{,}05 = -7{,}8$$

$$\textcircled{\bar{2}}\quad -7{,}8 \cdot 0{,}98 = -7{,}6$$
$$\cdot\, 0{,}02 = -0{,}2$$

„povratak” na okvir:

$$M_{6,7} = M_{7,6} = \frac{1}{2} M_{\bar{3},\mathrm{g}} = \frac{1}{2} \cdot (-148{,}4) = -74{,}2 \text{ kNm}$$

$$M_{6,3} = M_{7,4} = \frac{1}{2} M_{\bar{3},\bar{2}} = \frac{1}{2} \cdot 148{,}4 = 74{,}2 \text{ kNm}$$

$$M_{3,6} = M_{4,7} = \frac{1}{2} M_{\bar{2},\bar{3}} = \frac{1}{2} \cdot 151{,}6 = 75{,}8 \text{ kNm}$$

$$M_{3,1} = M_{4,2} = \frac{1}{2} M_{\bar{2},\bar{1}} = \frac{1}{2} \cdot 150{,}0 = 75{,}0 \text{ kNm}$$

$$M_{3,4} = 4\, k_{\{3,4\}}\, \varphi_3 + 2\, k_{\{3,4\}}\, \varphi_4 = 4\, k_{\{3,4\}}\, \varphi_{\bar{2}} + 2\, k_{\{3,4\}}\, \varphi_{\bar{2}} = 6\, k_{\{3,4\}}\, \varphi_{\bar{2}}$$

$$M_{4,3} = 2\, k_{\{3,4\}}\, \varphi_3 + 4\, k_{\{3,4\}}\, \varphi_4 = 6\, k_{\{3,4\}}\, \varphi_{\bar{2}}$$

$$M_{4,5} = 3\, k_{\{4,5\}}\, \varphi_4 = 3\, k_{\{4,5\}}\, \varphi_{\bar{2}}$$

$$M_{\bar{2},\mathrm{g}} = M_{3,4} + M_{4,3} + M_{4,5} = (2 \cdot 6\, k_{\{3,4\}} + 3\, k_{\{4,5\}})\, \varphi_{\bar{2}} = 30\, EI\, \varphi_{\bar{2}}$$

$$\varphi_{\bar{2}} = \frac{M_{\bar{2},\mathrm{g}}}{30\, EI}$$

$$M_{3,4} = 6\, k_{\{3,4\}}\, \frac{M_{\bar{2},\mathrm{g}}}{30\, EI} = \frac{6 \cdot 2\, EI}{30\, EI}\, M_{\bar{2},\mathrm{g}} = \frac{2}{5} M_{\bar{2},\mathrm{g}} = \frac{2}{5} \cdot (-301{,}6) = -120{,}6 \text{ kNm}$$

$$M_{4,3} = \frac{2}{5} M_{\bar{2},\mathrm{g}} = -120{,}6 \text{ kNm}$$

$$M_{4,5} = 3\, k_{\{4,5\}}\, \frac{M_{\bar{2},\mathrm{g}}}{30\, EI} = \frac{3 \cdot 2\, EI}{30\, EI}\, M_{\bar{2},\mathrm{g}} = \frac{1}{5} M_{\bar{2},\mathrm{g}} = -60{,}3 \rightarrow -60{,}4 \text{ kNm}$$

$(2 \cdot 120{,}6 + 60{,}3 = 301{,}5)$

razdjelni koeficijenti za Crossov postupak:

$$k_3 = 3\,k_{\{1,3\}} + 4\,k_{\{3,4\}} + 4\,k_{\{3,6\}} = 3\cdot\frac{2\,EI}{3} + 4\cdot 2\,EI + 4\cdot\frac{EI}{3} = \frac{34}{3}\,EI$$

$$\mu_{\{3,1\}} = \frac{3\,k_{\{1,3\}}}{k_3} = \frac{3\cdot\frac{2\,EI}{3}}{\frac{34}{3}\,EI} = \frac{3}{17} = 0{,}18$$

$$\mu_{\{3,4\}} = \frac{4\,k_{\{3,4\}}}{k_3} = \frac{4\cdot 2\,EI}{\frac{34}{3}\,EI} = \frac{12}{17} = 0{,}71 \quad\rightarrow\quad 0{,}70$$

$$\mu_{\{3,6\}} = \frac{4\,k_{\{3,6\}}}{k_3} = \frac{4\cdot\frac{EI}{3}}{\frac{34}{3}\,EI} = \frac{2}{17} = \underline{0{,}12} \qquad \left(\tfrac{3}{17}+\tfrac{12}{17}+\tfrac{2}{17} = 1\right)$$

$$1{,}01$$

$$k_4 = 3\,k_{\{2,4\}} + 4\,k_{\{3,4\}} + 3\,k_{\{4,5\}} + 4\,k_{\{4,7\}}$$
$$= 3\cdot\frac{2\,EI}{3} + 4\cdot 2\,EI + 3\cdot 2\,EI + 4\cdot\frac{EI}{3} = \frac{52}{3}\,EI$$

$$\mu_{\{4,2\}} = \frac{3\,k_{\{2,4\}}}{k_4} = \frac{3\cdot\frac{2\,EI}{3}}{\frac{52}{3}\,EI} = \frac{3}{26} = 0{,}12$$

$$\mu_{\{4,3\}} = \frac{4\,k_{\{3,4\}}}{k_4} = \frac{4\cdot 2\,EI}{\frac{52}{3}\,EI} = \frac{6}{13} = 0{,}46 \quad\rightarrow\quad 0{,}45$$

$$\mu_{\{4,5\}} = \frac{3\,k_{\{4,5\}}}{k_4} = \frac{3\cdot 2\,EI}{\frac{52}{3}\,EI} = \frac{9}{26} = 0{,}35$$

$$\mu_{\{4,7\}} = \frac{4\,k_{\{4,7\}}}{k_4} = \frac{4\cdot\frac{EI}{3}}{\frac{52}{3}\,EI} = \frac{1}{13} = \underline{0{,}08} \qquad \left(\tfrac{3}{26}+\tfrac{6}{13}+\tfrac{9}{26}+\tfrac{1}{13} = 1\right)$$

$$1{,}01$$

$$k_6 = 4\,k_{\{3,6\}} + 4\,k_{\{6,7\}} = 4\cdot\frac{EI}{3} + 4\cdot EI = \frac{16}{3}\,EI$$

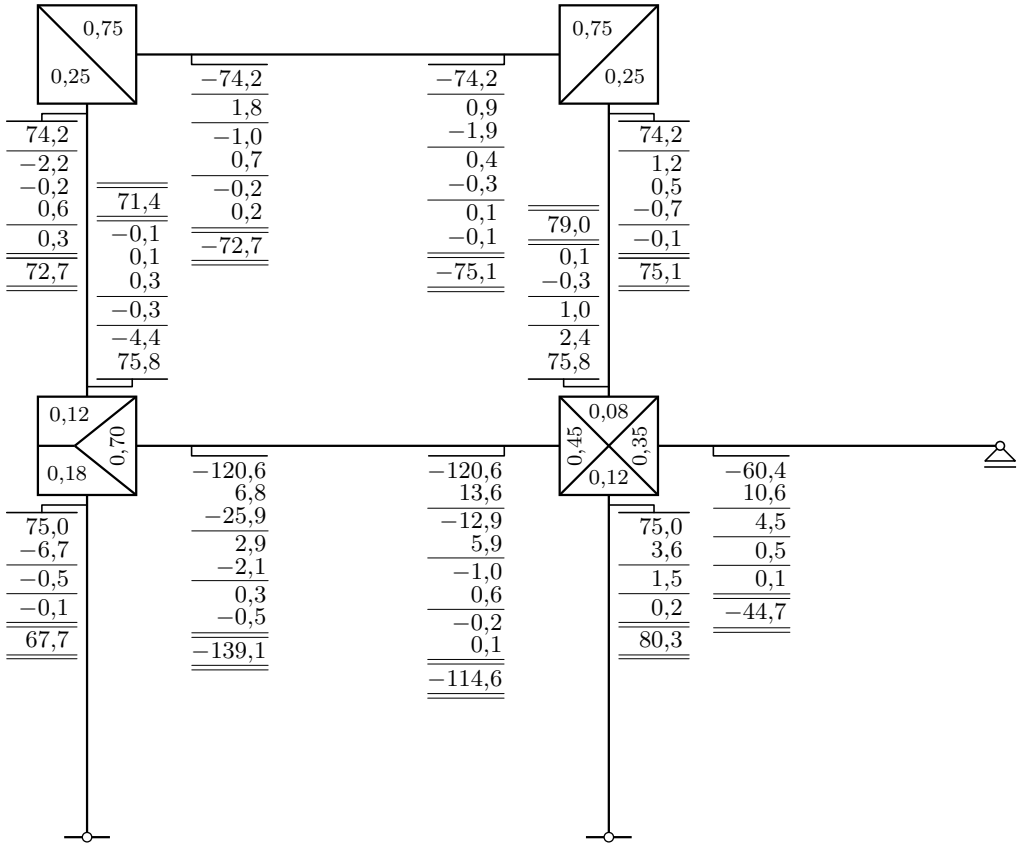
$$\mu_{\{6,3\}} = \frac{4\,k_{\{3,6\}}}{k_6} = \frac{4\cdot\frac{EI}{3}}{\frac{16}{3}\,EI} = \frac{1}{4} = 0{,}25$$

$$\mu_{\{6,7\}} = \frac{4\,k_{\{6,7\}}}{k_6} = \frac{4\cdot EI}{\frac{16}{3}\,EI} = \frac{3}{4} = 0{,}75$$

$$\mu_{\{7,4\}} = 0{,}25$$

$$\mu_{\{7,6\}} = 0{,}75$$

iteracija (relaksacija):



③ $75,0 + 75,8 - 120,6 = 30,2$

④ $75,0 + 75,8 - 120,6 - 60,4 = -30,2$

$30,2 \cdot 0,12 = 3,6$

$\cdot 0,45 = 13,6 \quad (\Rightarrow 6,8)$

$\cdot 0,35 = 10,6$

$\cdot 0,08 = 2,4 \quad (\Rightarrow 1,2)$

③ $30,2 + 6,8 = 37,0$

$-37,0 \cdot 0,18 = -6,7$

$\cdot 0,70 = -25,9$

$(\Rightarrow -12,95 \rightarrow -12,9)$

$\cdot 0,12 = -4,4 \quad (\Rightarrow -2,2)$

④ $12,9 \cdot 0,12 = 1,5$

$\cdot 0,45 = 5,8 \rightarrow 5,9$

$(\Rightarrow 2,95 \rightarrow 2,9)$

$\cdot 0,35 = 4,5$

$\cdot 0,08 = 1,0 \quad (\Rightarrow 0,5)$

$12,8$

③ $-2,9 \cdot 0,18 = -0,5$

$\cdot 0,70 = -2,0 \rightarrow -2,1$

$(\Rightarrow -1,05 \rightarrow -1,0)$

$\cdot 0,12 = -0,3 \quad (\Rightarrow -0,15 \rightarrow -0,2)$

$-2,8$

⑥ $-2,2 - 0,2 = -2,4$

$2,4 \cdot 0,25 = 0,6 \quad (\Rightarrow 0,3)$

$\cdot 0,75 = 1,8 \quad (\Rightarrow 0,9)$

⑦ $0,9 + 1,2 + 0,5 = 2,4$

$-2,6 \cdot 0,25 = -0,7 \quad (-0,65) \quad (\Rightarrow -0,3)$

$\cdot 0,75 = -1,9 \quad (-1,95) \quad (\Rightarrow -1,0)$

④ $-1,0 - 0,3 = -1,3$

$1,3 \cdot 0,12 = 0,2$

$\cdot 0,45 = 0,6 \quad (\Rightarrow 0,3)$

$\cdot 0,35 = 0,5$

$\cdot 0,08 = 0,1 \quad (\Rightarrow 0)$

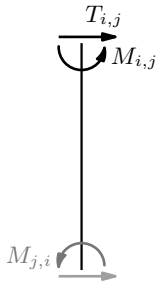
⑥ $1,0 \cdot 0,25 = 0,3 \quad (0,25) \quad (\Rightarrow 0,1)$

$\cdot 0,75 = 0,7 \quad (0,75) \quad (\Rightarrow 0,4)$

$$
\begin{aligned}
&\textcircled{7} \quad -0{,}4 \cdot 0{,}25 \;=\; -0{,}1 \qquad (\Rightarrow\; 0) \\
&\qquad\quad \cdot 0{,}75 \;=\; -0{,}3 \qquad (\Rightarrow\; -0{,}2)
\end{aligned}
$$

$$
\begin{aligned}
&\textcircled{4} \quad 0{,}2 \cdot 0{,}12 \;=\; 0 \\
&\qquad\quad \cdot 0{,}45 \;=\; 0{,}1 \qquad (\Rightarrow\; 0) \\
&\qquad\quad \cdot 0{,}35 \;=\; 0{,}1 \\
&\qquad\quad \cdot 0{,}08 \;=\; 0 \\
&\textcircled{6} \quad 0{,}2 \cdot 0{,}25 \;=\; 0 \\
&\qquad\quad \cdot 0{,}75 \;=\; 0{,}2 \qquad (\Rightarrow\; 0{,}1) \\
&\textcircled{7} \quad -0{,}1 \cdot 0{,}25 \;=\; 0 \\
&\qquad\quad \cdot 0{,}75 \;=\; -0{,}1 \qquad (\Rightarrow\; 0)
\end{aligned}
$$

vrijednosti poprečnih sila u stupovima:



$$-T_{i,j}\,h \;+\; M_{i,j} \;+\; M_{j,i} \;=\; 0$$

$$T'_{6,3} \;=\; \frac{M_{6,3} \;+\; M_{3,6}}{3} \;=\; \frac{72{,}7 \;+\; 71{,}4}{3} \;=\; 48{,}0 \text{ kN}$$

$$T'_{7,4} \;=\; \frac{M_{7,4} \;+\; M_{4,7}}{3} \;=\; \frac{75{,}1 \;+\; 79{,}0}{3} \;=\; 51{,}4 \text{ kN}$$

$$T'_{\bar{3},\bar{2}} \;=\; T'_{6,3} \;+\; T'_{7,4} \;=\; 48{,}0 \;+\; 51{,}4 \;=\; 99{,}4 \text{ kN} \qquad\qquad T_{\bar{3},\bar{2}} \;=\; 100 \text{ kN}$$

$$T'_{3,1} \;=\; \frac{M_{3,1}}{3} \;=\; \frac{67{,}7}{3} \;=\; 22{,}6 \text{ kN}$$

$$T'_{4,2} \;=\; \frac{M_{4,2}}{3} \;=\; \frac{80{,}3}{3} \;=\; 26{,}8 \text{ kN}$$

$$T'_{\bar{2},\bar{1}} \;=\; T'_{3,1} \;+\; T'_{4,2} \;=\; 22{,}6 \;+\; 26{,}8 \;=\; 49{,}4 \text{ kN} \qquad\qquad T_{\bar{3},\bar{2}} \;=\; 50 \text{ kN}$$

popravni koeficijent i konačne vrijednosti momenata:

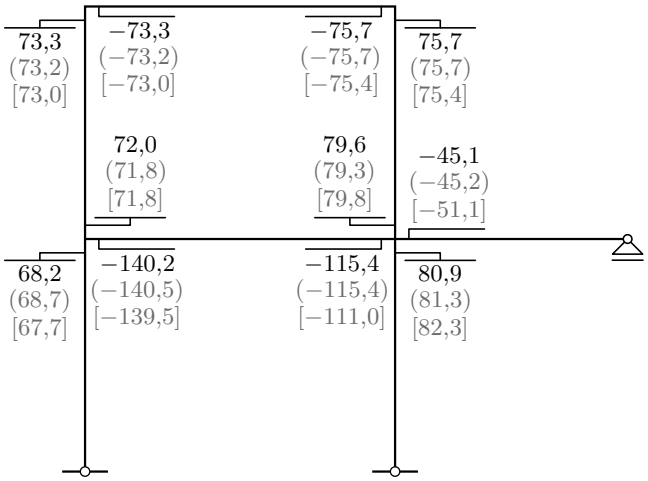
$$\left.\begin{aligned} T'_{\bar{3},\bar{2}} \;&<\; T_{\bar{3},\bar{2}} \\ T'_{\bar{2},\bar{1}} \;&<\; T_{\bar{2},\bar{1}} \end{aligned}\right\} \qquad \alpha \;>\; 1, \quad \alpha\, T'_{i,j} \;\simeq\; T_{i,j}$$

$$\alpha \;=\; \frac{\sum_i |T_{i,j}|\, h_i}{\sum_i |T'_{i,j}|\, h_i} \;=\; \frac{100 \cdot 3 \;+\; 50 \cdot 3}{99{,}4 \cdot 3 \;+\; 49{,}4 \cdot 3} \;=\; \frac{100 \;+\; 50}{99{,}4 \;+\; 49{,}4} \;=\; \frac{\sum_i |T_{i,j}|}{\sum_i |T'_{i,j}|} \;=\; \frac{150}{148{,}8} \;=\; 1{,}008$$

$$\alpha\, T'_{\bar{3},\bar{2}} \;=\; 1{,}008 \cdot 99{,}4 \;=\; 100{,}2 \text{ kN}$$

$$\alpha\, T'_{\bar{2},\bar{1}} \;=\; 1{,}008 \cdot 49{,}4 \;=\; 49{,}8 \text{ kN}$$

$$M \;=\; \alpha\, M^{(\text{Cr.})} \qquad\qquad \left[\text{ ili } \; M \;=\; M^{(1.\,\text{Cr.})} \;+\; \alpha\, M^{(2.\,\text{Cr.})}\right]$$



(u čvoru 4 je vrijednost −115,5 promijenjena u −115,4 kako bi se ostvarila ravnoteža)

(Vrijednosti navedene u oblim zagradama izračunane su inženjerskom, a vrijednosti navedene u uglatim zagradama općom metodom pomakā. Proračun je proveden računalnim programom DiM; kako bi se simulirala inženjerska metoda, ploštine su poprečnih presjeka pomnožene velikim brojem.)

dijagram momenata savijanja: DZ!