

## GS 2. — 28. kolovoza 2024.

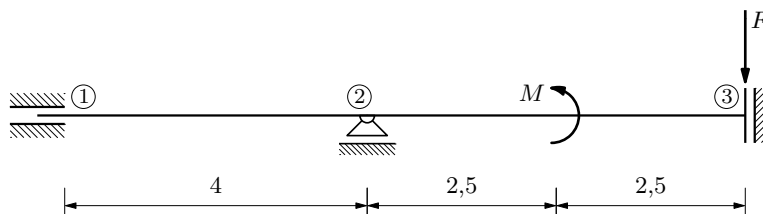
### Zadatak 2.b.

Pomoću utjecajne linije nacrtane relaksacijskim postupkom izračunajte vrijednost reaktivnoga momenta iznad srednjega ležaja!

$$EI = \text{const.}$$

$$F = 125 \text{ kN}$$

$$M = 75 \text{ kNm}$$



$$k_{1,2} = \frac{EI}{4}, \quad k_{2,3} = \frac{EI}{5}$$

nepoznanice za inženjersku metodu pomakā:  $\varphi_2$  &  $w_3$

uz statičku kondenzaciju pomaka  $\vec{w}_3$ :  $\varphi_2$

**relaksacija bez statičke kondenzacije:**

jedinični kut zaokreta neposredno lijevo od srednjega ležaja (zaokret kraja 2 štapa {1, 2}):

smisao vrtnje momenta:  jedinični kut: 

vrijednosti momenata upetosti:

$$\bar{M}_{1,2} = 2k_{\{1,2\}} \cdot (-1) = -\frac{EI}{2} \quad \& \quad \bar{M}_{2,1} = 4k_{\{1,2\}} \cdot (-1) = -EI$$

razdjelni koeficijenti:

$$k_2 = 4k_{1,2} + 4k_{2,3} = 4\frac{EI}{4} + 4\frac{EI}{5} = \frac{9}{5}EI$$

$$\mu_{2,1} = \frac{4k_{1,2}}{k_2} = \frac{4\frac{EI}{4}}{\frac{9EI}{5}} = \frac{5}{9}$$

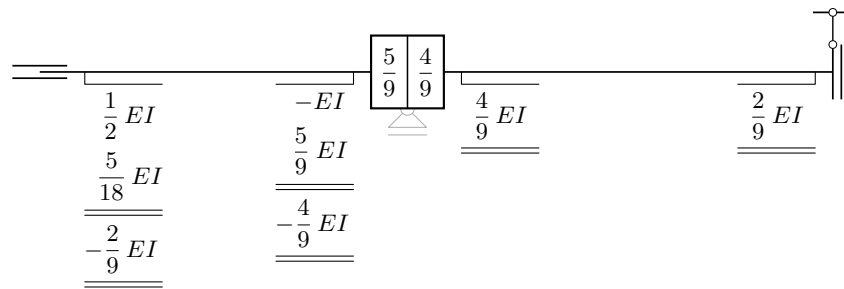
$$\mu_{2,3} = \frac{4k_{2,3}}{k_2} = \frac{4\frac{EI}{5}}{\frac{9EI}{5}} = \frac{4}{9}$$

$$\mu_{2,1} + \mu_{2,3} = \frac{5}{9} + \frac{4}{9} = 1$$

prijenosni koeficijenti:

$$2 \rightarrow 1 : \frac{1}{2}, \quad 2 \rightarrow 3 : \frac{1}{2}$$

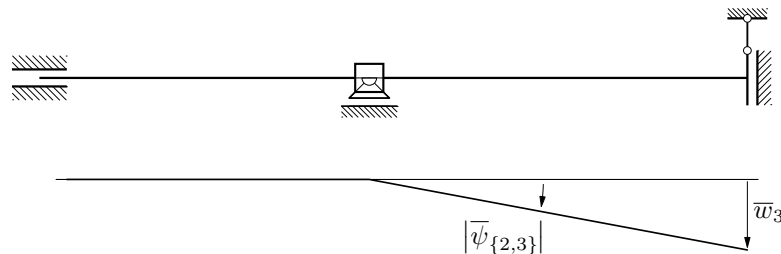
relaksacija bez iteracije:



reakcija u dodanom spoju:

$$R_1 = T_{3,2} = \frac{1}{\ell_{\{2,3\}}} (M_{2,3} + M_{3,2}) = \frac{1}{5} \left( \frac{4}{9} EI + \frac{2}{9} EI \right) = \frac{2}{15} EI$$

prisilni pomak:

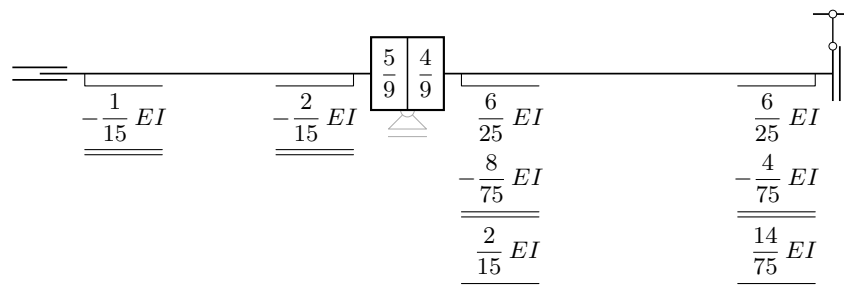


$$\bar{w}_3 = 1, \quad \bar{\psi}_{\{2,3\}} = -\frac{\bar{w}_3}{\ell_{\{2,3\}}} = -\frac{1}{5}$$

vrijednosti momenata upetosti:

$$\bar{M}_{2,3} = \bar{M}_{3,2} = -6 k_{\{2,3\}} \bar{\psi}_{\{2,3\}} = -6 \frac{EI}{5} \left( -\frac{1}{5} \right) = \frac{6}{25} EI$$

relaksacija bez iteracije:



reakcija u dodanom spoju:

$$R_2 = T_{3,2}^{(2)} = \frac{1}{5} \left( \frac{2}{15} EI + \frac{14}{75} EI \right) = \frac{8}{125} EI$$

$$R_1 + \varrho R_2 = 0 \quad \Rightarrow \quad \varrho = -\frac{R_1}{R_2} = -\frac{\frac{2}{15} EI}{\frac{8}{125} EI} = -\frac{25}{12}$$

konačne vrijednosti momenata:

$$M_{1,2} = M_{1,2}^{(1, \text{Cross})} + \varrho M_{1,2}^{(2, \text{Cross})} = -\frac{2}{9}EI - \frac{25}{12} \left( -\frac{1}{15}EI \right) = -\frac{1}{12}EI$$

$$M_{2,1} = -\frac{4}{9}EI - \frac{25}{12} \left( -\frac{2}{15}EI \right) = -\frac{1}{6}EI$$

$$M_{2,3} = \frac{4}{9} - \frac{25}{12} \cdot \frac{2}{15}EI = \frac{1}{6}EI$$

$$M_{3,2} = \frac{2}{9} - \frac{25}{12} \cdot \frac{14}{75}EI = -\frac{1}{6}EI$$

ili: jedinični kut zaokreta neposredno desno od srednjega ležaja (zaokret kraja 2 štapa {2,3}):  
domaća zabava!

ili: **relaksacija uz statičku kondenzaciju:**

jedinični kut zaokreta neposredno lijevo od srednjega ležaja (zaokret kraja 2 štapa {1,2}):

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razdjelni koeficijenti:

$$k_2 = 4k_{1,2} + k_{2,3} = 4\frac{EI}{4} + \frac{EI}{5} = \frac{6}{5}EI$$

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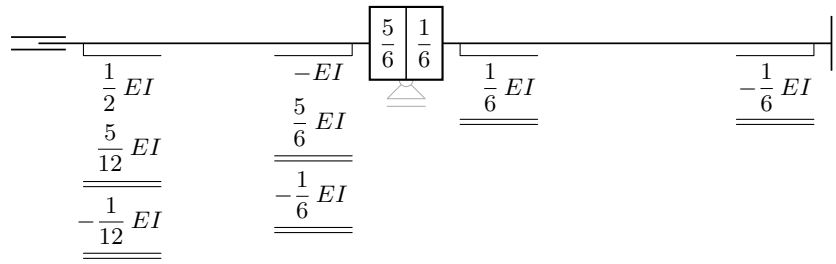
$$\mu_{2,3} = \frac{k_{2,3}}{k_2} = \frac{\frac{EI}{5}}{\frac{6EI}{5}} = \frac{1}{6}$$

$$\mu_{2,1} + \mu_{2,3} = \frac{5}{6} + \frac{1}{6} = 1$$

prijenosni koeficijenti:

$$2 \rightarrow 1 : \frac{1}{2}, \quad 2 \rightarrow 3 : -1$$

relaksacija bez iteracije:

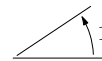


ili: jedinični kut zaokreta neposredno desno od srednjega ležaja (zaokret kraja 2 štapa {2,3}):

smisao vrtnje momenta:



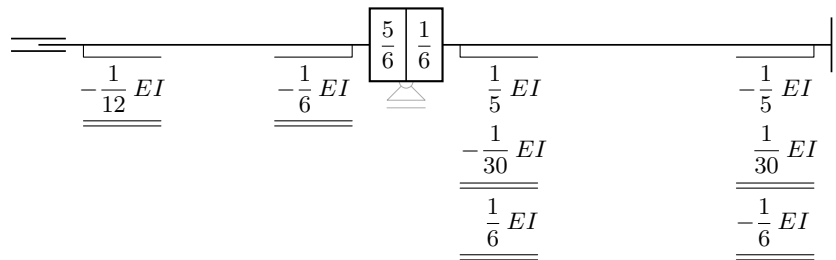
jedinični kut:



vrijednosti momenata upetosti:

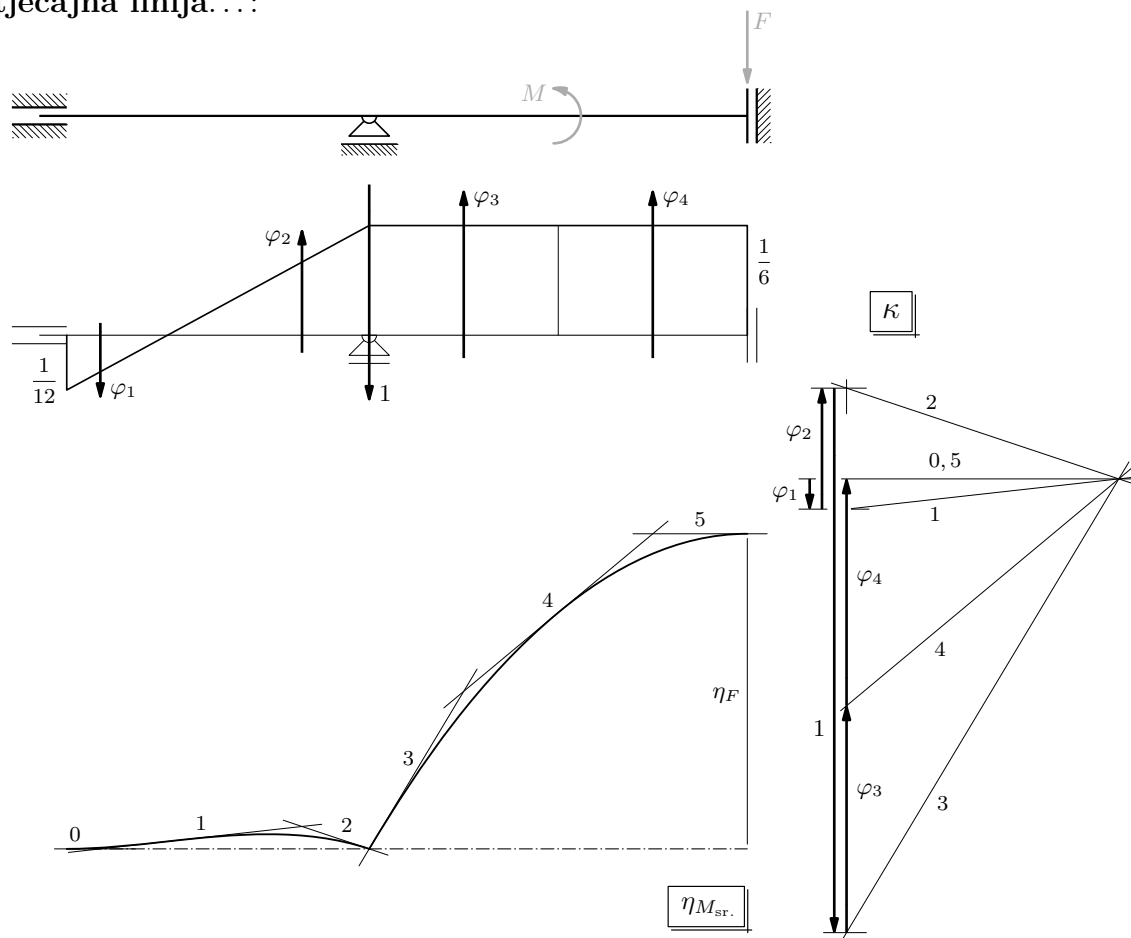
$$\bar{M}_{2,3} = k_{\{2,3\}} \cdot 1 = \frac{EI}{5} \quad \& \quad \bar{M}_{3,2} = -k_{\{2,3\}} \cdot 1 = -\frac{EI}{5}$$

relaksacija bez iteracije:



i, na kraju, na sljedećoj stranici, utjecajna linija...

utjecajna linija. . . :



mjerilo duljina: 1 cm :: 1 m

$$\varphi_1 = \frac{1}{2} \cdot \frac{1}{12} \cdot \left( \frac{1}{3} \cdot 4 \right) = \frac{1}{18}$$

$$\varphi_2 = \frac{1}{2} \cdot \frac{1}{6} \cdot \left( \frac{2}{3} \cdot 4 \right) = \frac{2}{9}$$

$$\varphi_3 = \varphi_4 = \frac{1}{6} \cdot \left( \frac{1}{2} \cdot 5 \right) = \frac{5}{12}$$

$$\text{provjera: } \frac{1}{18} - \frac{2}{9} + 1 - \frac{5}{12} - \frac{5}{12} = 0 \quad [\text{zašto?}]$$

mjerilo kutova: 1 cm ::  $\frac{5}{36}$

$$\tilde{\varphi}_1 = \frac{2}{5} = 0,4 \text{ cm,}$$

$$\tilde{\varphi}_2 = \frac{8}{5} = 1,6 \text{ cm,}$$

$$\tilde{\varphi}_3 = \tilde{\varphi}_4 = 3 \text{ cm,}$$

$$\tilde{1} = \frac{36}{5} = 7,2 \text{ cm}$$

$$\chi = \frac{1}{2} \Rightarrow \tilde{\chi} = 3,6 \text{ cm}$$

... i primjena:

očitano:  $|\tilde{\eta}_F| = 41 \text{ i } 2/3 \text{ mm} \simeq 4,17 \text{ cm}$

$$\chi = \frac{1}{2} \quad \Rightarrow \quad n = 2$$

$$|\eta_F| = \frac{m}{n} \tilde{\eta}_F = \frac{1}{2} \cdot 4,17 = 2,085, \quad \eta_F = -2,085$$

nagib tangente na  $\eta_{M_d}$  u hvatištu momenta:

$$\text{tg } \alpha_M = -\varphi_4 = -\frac{5}{12} = -0,417$$

$$M_s. = F \eta_F + M (-\text{tg } \alpha_M) = 125 \cdot (-2,085) + 75 \cdot (-(-0,417)) = -229,35 \text{ kNm}$$

(smisao vrtnje je suprotan od pretpostavljenoga)