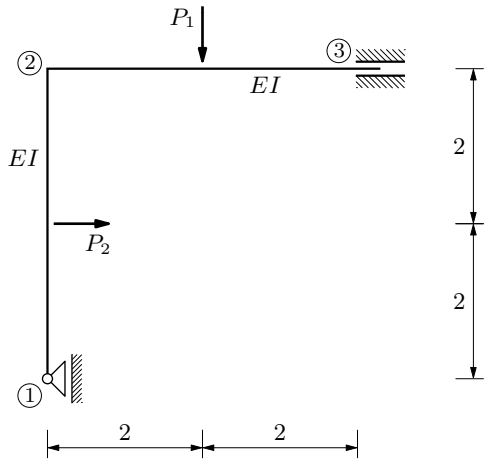
GS 2. — 6. veljače 2024.

Zadatak 4.

Relaksacijskim postupkom nacrtajte momentni dijagram! Provjerite rezultat!

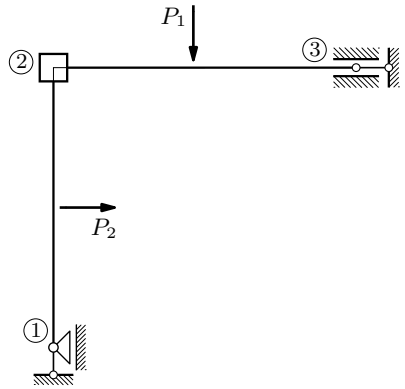
$P_1 = 125\ \text{kN}$

$P_2 = 50\ \text{kN}$



prvi korak: Crossov postupak na nepomičnom sistemu:

vrijednosti momenata upetosti:



$$\overline{M}^{(c)}_{2,1} = \overline{M}_{2,1} - \frac{1}{2}\overline{M}_{1,2} = -\frac{P_2\,\ell_{\{1,2\}}}{8} - \frac{1}{2}\frac{P_2\,\ell_{\{1,2\}}}{8}$$

$$= -\frac{3\,P_2\,\ell_{\{1,2\}}}{16} = -\frac{3\cdot 50\cdot 4}{16} = -37{,}5\ \text{kNm}$$

$$\overline{M}_{2,3} = \overline{M}_{3,2} = \frac{P_1\,\ell_{\{2,3\}}}{8} = \frac{125\cdot 4}{8} = 62{,}5\ \text{kNm}$$

razdjelni koeficijenti:

$k_{\{1,2\}} = k_{\{2,3\}} = k$

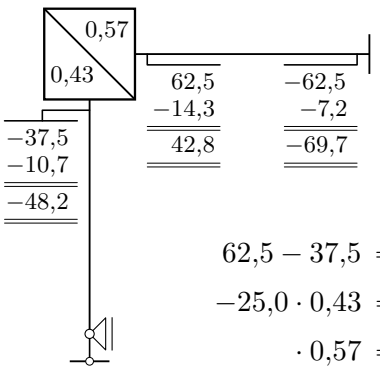
$k_2 = 3\,k + 4\,k = 7\,k$

$$\mu_{3,1} = \frac{3\,k}{k_2} = \frac{3}{7} = 0{,}43$$

$$\mu_{3,2} = \frac{4\,k}{k_2} = \frac{4}{7} = 0{,}57$$

$\mu_{3,1} + \mu_{3,2} = 0{,}43 + 0{,}57 = 1$

raspodjela i prijenos momenata:



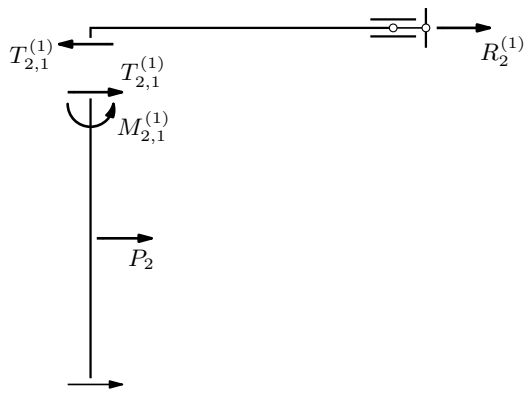
$$62{,}5 - 37{,}5 = 25{,}0$$

$$-25{,}0\cdot 0{,}43 = -10{,}7$$

$$\cdot\,0{,}57 = \underline{-14{,}3}$$

$$-25{,}0$$

vrijednosti reakcija u zamišljenim spojevima:



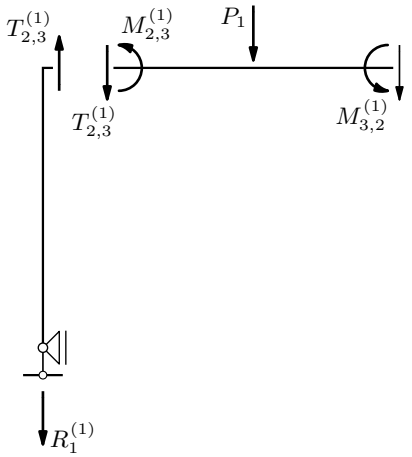
$$-T_{2,1}^{(1)} + R_2^{(1)} = 0$$

$$R_2^{(1)} = T_{2,1}^{(1)}$$

$$-4 \cdot T_{2,1}^{(1)} + M_{2,1}^{(1)} - 2 \cdot P_2 = 0$$

$$\begin{aligned} T_{2,1}^{(1)} &= \frac{M_{2,1}^{(1)}}{4} - \frac{P_2}{2} \\ &= \frac{-48{,}2}{4} - \frac{50}{2} \\ &= -37{,}1\,\text{kN} \end{aligned}$$

$$R_2^{(1)} = -37{,}1\,\text{kN}$$



$$-T_{2,3}^{(1)} + R_1^{(1)} = 0$$

$$R_1^{(1)} = T_{2,3}^{(1)}$$

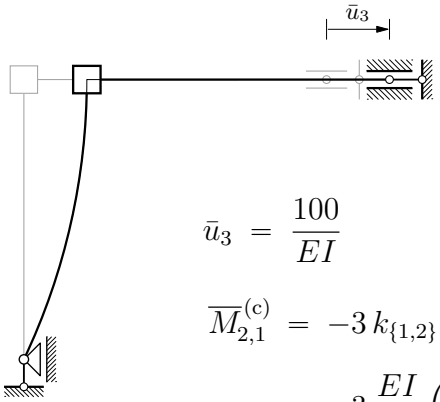
$$4 \cdot T_{2,3}^{(1)} + M_{2,3}^{(1)} + M_{3,2}^{(1)} + 2 \cdot P_1 = 0$$

$$\begin{aligned} T_{2,3}^{(1)} &= -\frac{M_{2,3}^{(1)} + M_{3,2}^{(1)}}{4} - \frac{P_1}{2} \\ &= -\frac{48{,}2 - 69{,}7}{4} - \frac{125}{2} \\ &= -57{,}1\,\text{kN} \end{aligned}$$

$$R_1^{(1)} = -57{,}1\,\text{kN}$$

drugi korak: proširenje Crossova postupka:

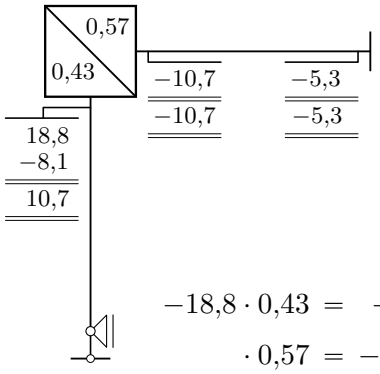
vrijednost momenta upetosti:



$$\bar{u}_3 = \frac{100}{EI}$$

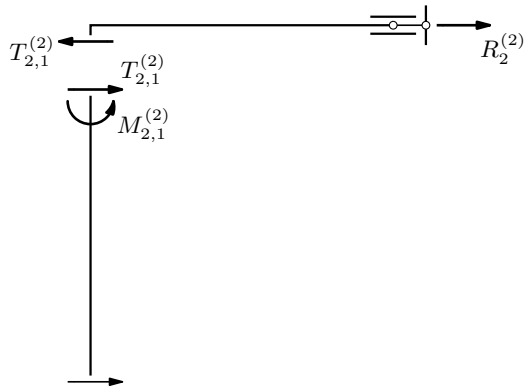
$$\begin{aligned} \overline{M}_{2,1}^{(c)} &= -3\, k_{\{1,2\}}\, \overline{\psi}_{\{1,2\}} \\ &= -3\, \frac{EI}{4} \left(-\frac{\bar{u}_3}{4}\right) \\ &= 3 \cdot \frac{EI}{4} \cdot \frac{\frac{100}{EI}}{4} \\ &= 18{,}8\,\text{kNm} \end{aligned}$$

raspodjela i prijenos momenata:



$$\begin{aligned} -18{,}8 \cdot 0{,}43 &= -8{,}1 \\ \cdot\, 0{,}57 &= -10{,}7 \\ &\overline{-18{,}8} \end{aligned}$$

vrijednosti reakcija u zamišljenim spojevima:

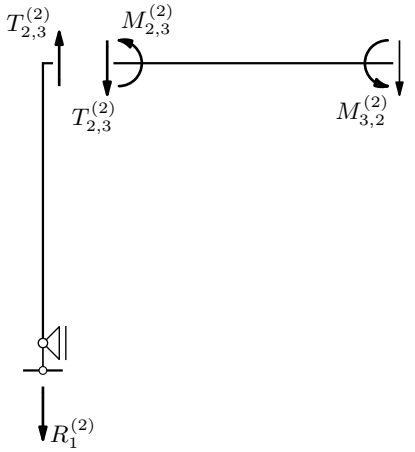


$$-T_{2,1}^{(2)} + R_2^{(2)} = 0$$

$$-4 \cdot T_{2,1}^{(2)} + M_{2,1}^{(2)} = 0$$

$$T_{2,1}^{(2)} = \frac{M_{2,1}^{(2)}}{4} = \frac{10{,}7}{4} = 2{,}7\,\text{kN}$$

$$R_2^{(2)} = 2{,}7\,\text{kN}$$



$$-T_{2,3}^{(2)} + R_1^{(2)} = 0$$

$$4 \cdot T_{2,3}^{(2)} + M_{2,3}^{(2)} + M_{3,2}^{(2)} = 0$$

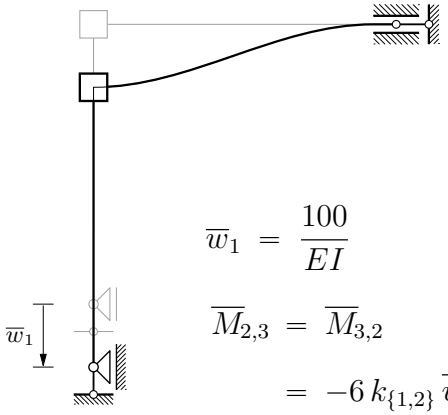
$$T_{2,3}^{(2)} = -\frac{M_{2,3}^{(2)} + M_{3,2}^{(2)}}{4}$$

$$= -\frac{-10{,}7 - 5{,}3}{4} = 4{,}0\,\text{kN}$$

$$R_1^{(2)} = 4{,}0\,\text{kN}$$

treći korak: proširenje Crossova postupka:

vrijednosti momenta upetosti:



$$\overline{w}_1 = \frac{100}{EI}$$

$$\overline{M}_{2,3} = \overline{M}_{3,2}$$

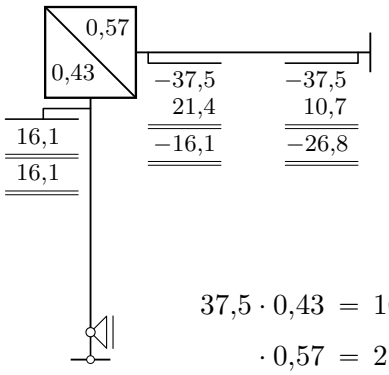
$$= -6\, k_{\{1,2\}}\, \overline{\psi}_{\{2,3\}}$$

$$= -6\, \frac{EI}{4}\, \frac{\overline{w}_1}{4}$$

$$= -6 \cdot \frac{EI}{4} \cdot \frac{\frac{100}{EI}}{4}$$

$$= -37{,}5\,\text{kNm}$$

raspodjela i prijenos momenata:

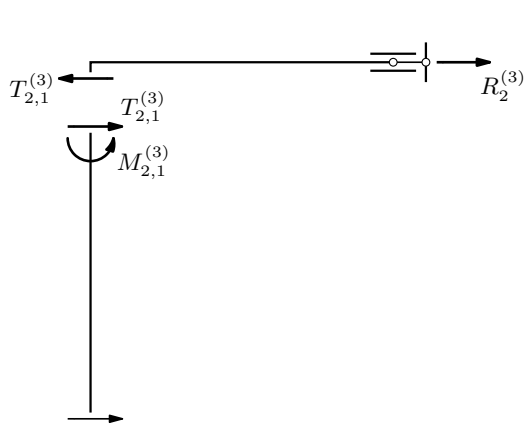


$$37{,}5 \cdot 0{,}43 = 16{,}1$$

$$\cdot\, 0{,}57 = \underline{21{,}4}$$

$$37{,}5$$

vrijednosti reakcija u zamišljenim spojevima:

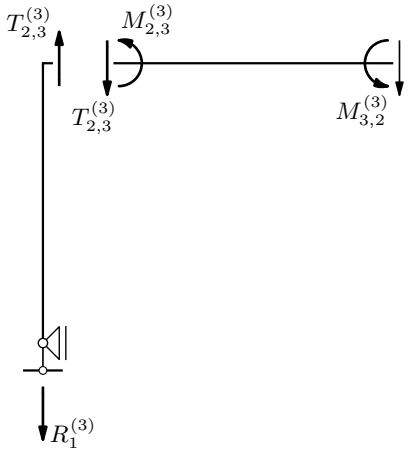


$$-T_{2,1}^{(3)} + R_2^{(3)} = 0$$

$$-4 \cdot T_{2,1}^{(3)} + M_{2,1}^{(3)} = 0$$

$$T_{2,1}^{(3)} = \frac{M_{2,1}^{(3)}}{4} = \frac{16{,}1}{4} = 4{,}0\,\mathrm{kN}$$

$$R_2^{(3)} = 4{,}0\,\mathrm{kN}$$



$$-T_{2,3}^{(3)} + R_1^{(3)} = 0$$

$$4 \cdot T_{2,3}^{(3)} + M_{2,3}^{(3)} + M_{3,2}^{(3)} = 0$$

$$T_{2,3}^{(3)} = -\frac{M_{2,3}^{(3)} + M_{3,2}^{(3)}}{4}$$

$$= -\frac{-16{,}1 - 26{,}8}{4} = 10{,}7\,\mathrm{kN}$$

$$R_1^{(3)} = 10{,}7\,\mathrm{kN}$$

. . . i, na kraju:

$$R_1^{(1)} + R_1^{(2)}\,\varrho_1 + R_1^{(3)}\,\varrho_2 = 0$$

$$R_2^{(1)} + R_2^{(2)}\,\varrho_1 + R_2^{(3)}\,\varrho_2 = 0$$

$$-57{,}1 + 4{,}0\,\varrho_1 + 10{,}7\,\varrho_2 = 0$$

$$-37{,}1 + 2{,}7\,\varrho_1 + 4{,}0\,\varrho_2 = 0$$

$$\varrho_1 = 13{,}08 \qquad \& \qquad \varrho_2 = 0{,}45$$

konačne vrijednosti momenata:

$$M_{i,j} = M_{i,j}^{(1)} + \varrho_1\,M_{i,j}^{(2)} + \varrho_2\,M_{i,j}^{(3)}$$

$$M_{2,1} = -48{,}2 + 13{,}08 \cdot 10{,}7 + 0{,}45 \cdot 16{,}1 = 99{,}0\,\mathrm{kNm}$$

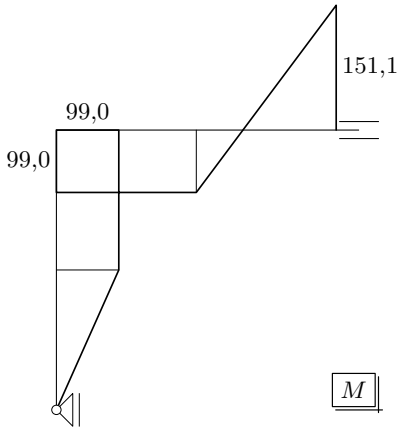
$$M_{2,3} = 48{,}2 + 13{,}08 \cdot (-10{,}7) + 0{,}45 \cdot (-16{,}1) = -99{,}0\,\mathrm{kNm}$$

$$M_{3,2} = -69{,}7 + 13{,}08 \cdot (-5{,}3) + 0{,}45 \cdot (-26{,}8) = -151{,}1\,\mathrm{kNm}$$

u horizontalnom štapu nema uzdužne sile, pa u vertikalnom štapu između čvora 2 i hvatišta sile P_2 nema poprečne sile, a moment se ne mijenja

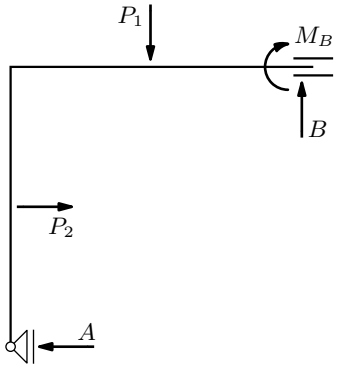
uzdužne sile nema ni u vertikalnom štapu, pa u horizontalnom štapu između čvora 2 i hvatišta sile P_1 nema poprečne sile, a moment se ne mijenja

dijagram momenata:



provjera:

sistem je statički određen



$$\sum F_x = 0: \qquad P_2 - A = 0 \qquad \Rightarrow \qquad A = P_2 = 50\,\text{kN}$$

$$\sum F_z = 0: \qquad P_1 - B = 0 \qquad \Rightarrow \qquad B = P_1 = 125\,\text{kN}$$

$$\sum M_{/B} = 0: \qquad -A \cdot 4 + P_2 \cdot 2 + P_1 \cdot 2 - M_B = 0$$

$$\Rightarrow \qquad M_B = -A \cdot 4 + P_2 \cdot 2 + P_1 \cdot 2 = 150\,\text{kNm}$$

$M_{h/2} = A \cdot 2 = 100\,\text{kNm}$ (u hvatištu sile P_2)

A i P_2 tvore spreg, pa se iznad hvatišta do čvora 2 moment ne mijenja: $M_{2,\text{dolje}} = M_{h,2}$

ravnoteža čvora 2: $M_{2,\text{dolje}} = M_{2,\text{desno}}$

moment se u horizontalnom štapu do hvatišta sile P_1 ne mijenja

iz svega slijedi da je dijagram momenata isti kao dijagram na vrhu stranice, s neznatno drukčijim (i, u stvari, točnijim) vrijednostima (u Crossovom smo postupku zaokruživanjem na jednu decimalu izgubili točnost)