GS 2. — 6. veljače 2024.

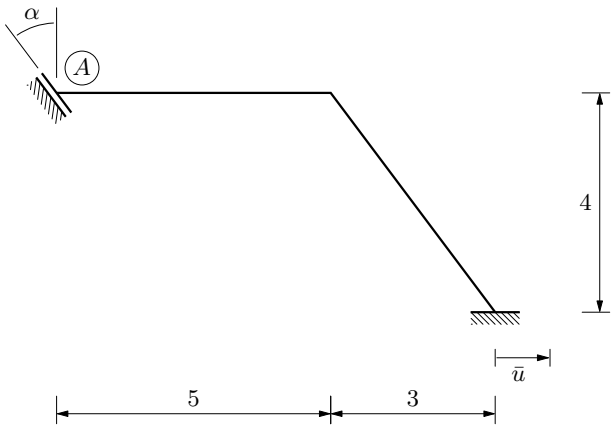
Zadatak 1.

Inženjerskom metodom pomakā nacrtajte dijagrame unutarnjih sila! Izračunajte duljinu pomaka točke A!

$\operatorname{tg}\alpha \;=\; {}^{3}\!/_{4}$

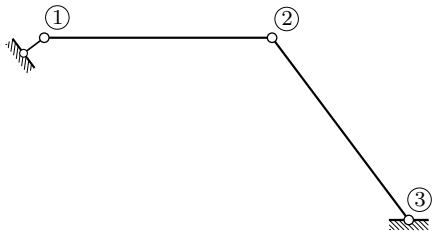
$EI \;=\; 162\,000\ \mathrm{kNm}^2$

$\bar{u} \;=\; 2\ \mathrm{mm}$

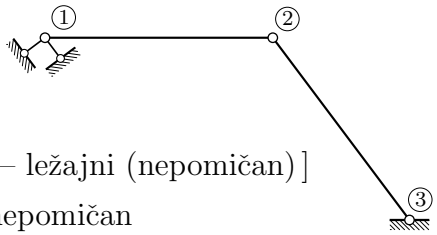


> Lipe pred crkvom istom sam nagađao u ogromnim konturama crnih neodređenih masa što su se u tamnoj tamnini još tamnije od ostaloga valovlja tmine za sebe kočile i talasale.
>
> K. S. Gjalski: *Notturno*

zglobna shema i kinematička analiza zglobne sheme (dodavanje spoj(ev)a s podlogom):



$s_{\min} \;=\; 2 \cdot 2 - 3 \;=\; 1$

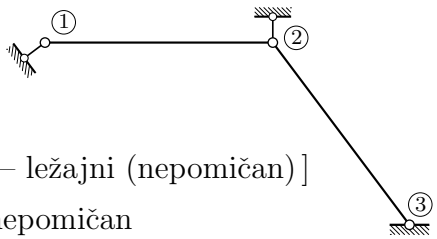


[čvor 3 – ležajni (nepomičan)]

čvor 1 nepomičan

$\rightarrow$ čvor 2 nepomičan

ili



[čvor 3 – ležajni (nepomičan)]

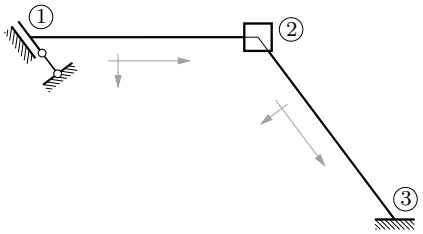
čvor 2 nepomičan

$\rightarrow$ čvor 1 nepomičan

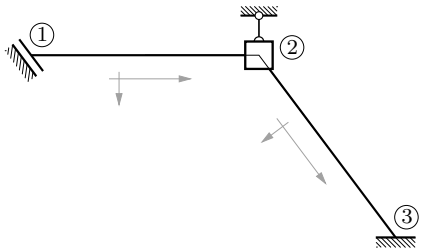
(spoj s podlogom u čvoru 2 može biti na bilo kojem pravcu osim osi štapa $\{2, 3\}$)

$\Rightarrow \quad s \;=\; s_{\min} \;=\; 1$

osnovni sistem za inženjersku metodu pomakā:



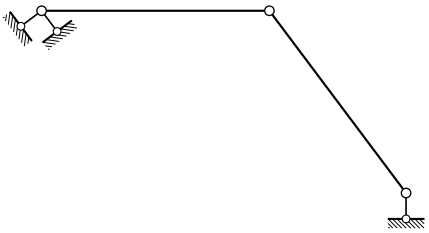
ili



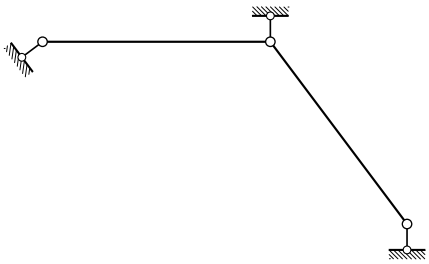
nepoznanice: φ_2 i δ_1 (ili u_1 ili w_1)

nepoznanice: φ_2 i w_2 (ili u_2 ili . . .)

mehanizam za utjecaj prisilnoga pomaka ležaja:

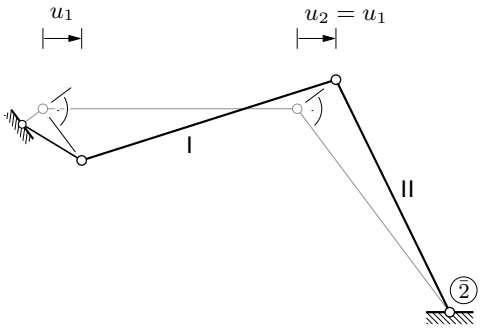


ili

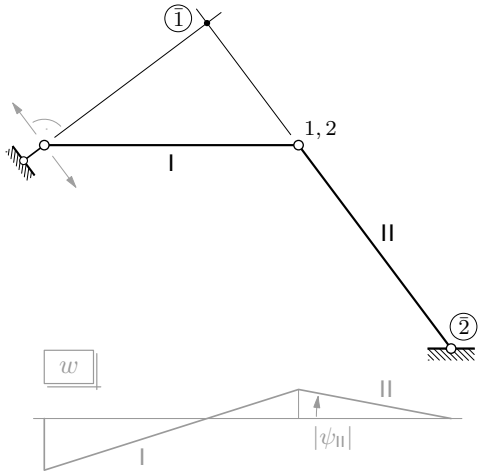


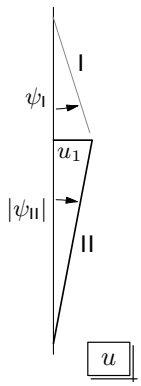
rješavanje pomoću lijevoga osnovnog sistema:

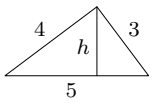
zôra radi, plan pomakā za neovisni translacijski pomak $\vec{u}_1$ (horizontalnu komponentu pomaka ležaja po kosom pravcu):



izrazi za kutove $\psi_{i,j}$ kao funkcije neovisnoga translacijskog pomaka u_1 (izvedeni pomoću dijagramā projekcijā pomakā na vertikalnu os):







$$\frac{h}{3} = \frac{4}{5}$$

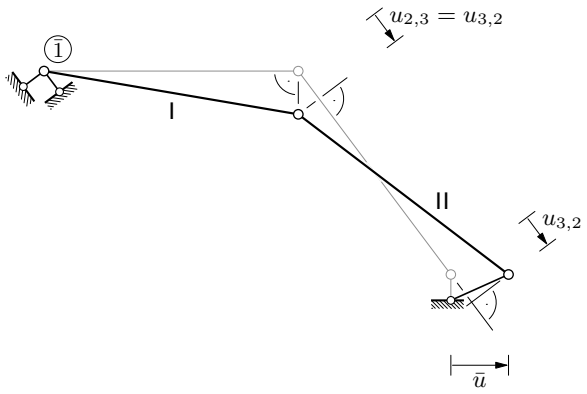
$$h = \frac{12}{5}$$

$$\psi_{\text{I}} = \frac{u_1}{h} = \frac{5}{12}\,u_1 = \psi_{\{1,2\}}$$

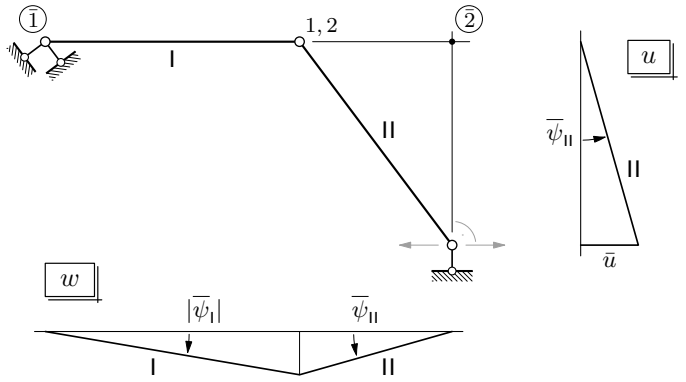
$$\psi_{\text{II}} = -\frac{u_1}{4} = \psi_{\{2,3\}}$$

(potpunosti radi, skiciran je i dijagram projekcija pomakā na vertikalnu os)

zôra radi, plan pomakā za prisilni pomak ležaja:



izračunavanje kutova $\overline{\psi}_{i,j}$ pomoću dijagramā projekcijā pomakā na horizontalnu i na vertikalnu os:



$$\overline{\psi}_{\text{II}} \;=\; \frac{\bar{u}}{4} \;=\; \overline{\psi}_{\{2,3\}}$$

$$\overline{\psi}_{\text{II}} \cdot 3 \;=\; |\overline{\psi}_{\text{I}}| \cdot 5$$

$$\overline{\psi}_{\text{I}} \;=\; -\frac{3}{5}\,\overline{\psi}_{\text{II}}$$

$$=\; -\frac{3}{20}\,\bar{u} \;=\; \overline{\psi}_{\{1,2\}}$$

vrijednosti momenata upetosti:

$$\overline{M}_{1,2} \;=\; \overline{M}_{2,1} \;=\; -6\,k_{\{1,2\}}\,\overline{\psi}_{\{1,2\}} \;=\; -6\;\frac{EI}{5}\left(-\frac{3}{20}\,\bar{u}\right)$$

$$=\; 6 \cdot \frac{162\,000}{5} \cdot 0{,}15 \cdot 0{,}002 \;=\; 58{,}32\,\text{kNm}$$

$$\overline{M}_{2,3} \;=\; \overline{M}_{3,2} \;=\; -6\,k_{\{2,3\}}\,\overline{\psi}_{\{2,3\}} \;=\; -6\,\frac{EI}{5}\,\frac{\bar{u}}{4} = \; -6 \cdot \frac{162\,000}{5} \cdot 0{,}25 \cdot 0{,}002 \;=\; -97{,}2\,\text{kNm}$$

(izrazi za) vrijednosti momenata savijanja na krajevima štapova:

$$M_{1,2} \;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} \;+\; \overline{M}_{1,2}$$

$$=\; 2 \cdot 32\,400\,\varphi_2 \;-\; 6 \cdot 32\,400 \cdot \frac{5}{12}\,u_1 \;+\; 58{,}32 \;=\; 64\,800\,\varphi_2 \;-\; 81\,000\,u_1 \;+\; 58{,}32$$

$$M_{2,1} \;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} \;+\; \overline{M}_{2,1}$$

$$=\; 4 \cdot 32\,400\,\varphi_2 \;-\; 6 \cdot 32\,400 \cdot \frac{5}{12}\,u_1 \;+\; 58{,}32 \;=\; 129\,600\,\varphi_2 \;-\; 81\,000\,u_1 \;+\; 58{,}32$$

$$M_{2,3} \;=\; 4\,k_{\{2,3\}}\,\varphi_2 \;-\; 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} \;+\; \overline{M}_{2,3}$$
$$\;=\; 4\cdot 32\,400\,\varphi_2 \;-\; 6\cdot 32\,400\left(-\frac{u_1}{4}\right) \;-\; 97{,}2 \;=\; 129\,600\,\varphi_2 \;+\; 48\,600\,u_1 \;-\; 97{,}2$$

$$M_{3,2} \;=\; 2\,k_{\{2,3\}}\,\varphi_2 \;-\; 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} \;+\; \overline{M}_{2,3}$$
$$\;=\; 2\cdot 32\,400\,\varphi_2 \;-\; 6\cdot 32\,400\left(-\frac{u_1}{4}\right) \;-\; 97{,}2 \;=\; 64\,800\,\varphi_2 \;+\; 48\,600\,u_1 \;-\; 97{,}2$$

jednadžba ravnoteže momenata u čvoru 2:

$$-M_{2,1} \;+\; (-M_{2,3}) \;=\; 0 \quad \Big| \quad \times(-1)$$

$$\big[129\,600\,\varphi_2 \;-\; 81\,000\,u_1 \;+\; 58{,}32\big] \;+\; \big[129\,600\,\varphi_2 \;+\; 48\,600\,u_1 \;-\; 97{,}2\big] \;=\; 0$$

$$259\,200\,\varphi_2 \;-\; 32\,400\,u_1 \;-\; 38{,}88 \;=\; 0$$

jednadžba rada na virtualnim pomacima:

(„vodeći" je virtualni pomak pomak $\delta\vec{u}_1$ koji odgovara neovisnom translacijskom pomaku $\vec{u}_1$, pa je dijagram projekcija (virtualnih) pomaka na horizontalnu os jednak dijagramu prikazanom na najdonjoj slici na stranici 2, uz zamjene $u_1 \to \delta u_1$ i $\psi_{\text{i}} \to \delta\psi_{\text{i}}$, a veza je kutova $\delta\psi_{\{i,j\}}$ i orijentirane duljine δu_1 ista kao veza $\psi_{\{i,j\}}$ i u_1:

$$\delta\psi_{\{1,2\}} \;=\; 5\,\delta u_1/12 \qquad \& \qquad \delta\psi_{\{2,3\}} \;=\; -\delta u_1/4\,)$$

$$\big(M_{1,2} \;+\; M_{2,1}\big)\,\delta\psi_{\{1,2\}} \;+\; \big(M_{2,3} \;+\; M_{3,2}\big)\,\delta\psi_{\{2,3\}} \;=\; 0$$

$$\big(64\,800\,\varphi_2 \;-\; 81\,000\,u_1 \;+\; 58{,}32 \;+\; 129\,600\,\varphi_2 \;-\; 81\,000\,u_1 \;+\; 58{,}32\big)\cdot\left(\frac{5}{12}\,\delta u_1\right)$$
$$+\; \big(129\,600\,\varphi_2 \;+\; 48\,600\,u_1 \;-\; 97{,}2 \;+\; 64\,800\,\varphi_2 \;+\; 48\,600\,u_1 \;-\; 97{,}2\big)\cdot\left(-\frac{\delta u_1}{4}\right) \;=\; 0$$

$$\big(32\,400\,\varphi_2 \;-\; 91\,800\,u_1 \;+\; 72{,}9\big)\,\delta u_1 \;=\; 0 \quad \forall\,\delta u_1$$

$$32\,400\,\varphi_2 \;-\; 91\,800\,u_1 \;+\; 72{,}9 \;=\; 0 \quad \Big| \quad \times(-1)$$

$$-32\,400\,\varphi_2 \;+\; 91\,800\,u_1 \;-\; 72{,}9 \;=\; 0$$

sustav jednadžbi i njegovo rješenje:

$$259\,200\,\varphi_2 \;-\; 32\,400\,u_1 \;=\; 38{,}88$$

$$-32\,400\,\varphi_2 \;+\; 91\,800\,u_1 \;=\; 97{,}2 \qquad \text{(sustav je simetričan)}$$

$$\varphi_2 \;=\; 0{,}000\,295\,385 \qquad \& \qquad u_1 \;=\; 0{,}001\,163\,08\ \text{m}$$

vrijednosti momenata savijanja na krajevima štapova:

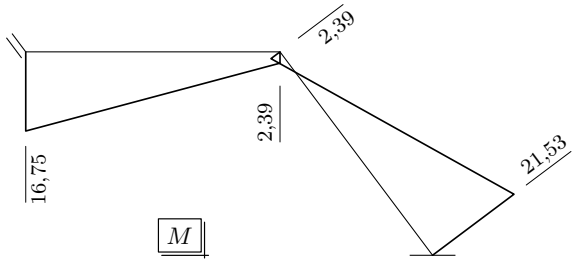
$$M_{1,2} = 64\,800 \cdot 0{,}000\,295\,385 - 81\,000 \cdot 0{,}001\,163\,08 + 58{,}32 = -16{,}748\,5\ \text{kNm}$$

$$M_{2,1} = 129\,600 \cdot 0{,}000\,295\,385 - 81\,000 \cdot 0{,}001\,163\,08 + 58{,}32 = 2{,}392\,42\ \text{kNm}$$

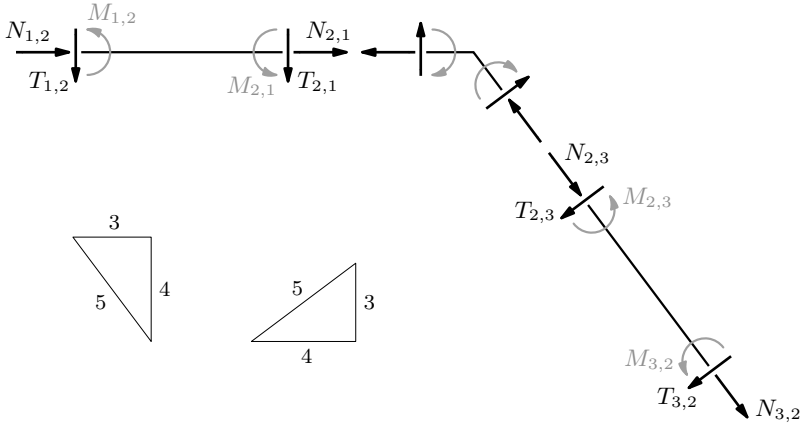
$$M_{2,3} = 129\,600 \cdot 0{,}000\,295\,385 + 48\,600 \cdot 0{,}001\,163\,08 - 97{,}2 = -2{,}392\,42\ \text{kNm}$$

$$M_{3,2} = 64\,800 \cdot 0{,}000\,295\,385 + 48\,600 \cdot 0{,}001\,163\,08 - 97{,}2 = -21{,}533\,4\ \text{kNm}$$

dijagram momenata savijanja:



vrijednosti poprečnih sila:



$$T_{1,2} \cdot 5 + M_{1,2} + M_{2,1} = 0 \qquad \text{(ravnoteža momenata u odnosu na kraj 2 štapa } \{1,2\}\text{)}$$

$$T_{1,2} = -\frac{M_{1,2} + M_{2,1}}{5} = -\frac{-16{,}748\,5 + 2{,}392\,42}{5} = 2{,}871\,22\ \text{kN} = -T_{(\xi_{(1,2)}=0^+)}$$

$$T_{1,2} + T_{2,1} = 0 \qquad \text{(ravnoteža sila okomitih na os štapa } \{1,2\}\text{)}$$

$$T_{2,1} = -T_{1,2} = -2{,}871\,22\ \text{kN} = T_{(\xi_{(1,2)}=5^-)}$$

$$T_{2,3} \cdot 5 + M_{2,3} + M_{3,2} = 0 \qquad \text{(ravnoteža momenata u odnosu na kraj 3 štapa } \{2,3\}\text{)}$$

$$T_{2,3} = -\frac{M_{2,3} + M_{3,2}}{5} = -\frac{-2{,}392\,42 - 21{,}533\,4}{5} = 4{,}785\,16\ \text{kN} = -T_{(\xi_{(2,3)}=0^+)}$$

$$T_{2,3} + T_{3,2} = 0 \qquad \text{(ravnoteža sila okomitih na os štapa } \{2,3\}\text{)}$$

$$T_{3,2} = -T_{2,3} = -4{,}785\,16\ \text{kN} = T_{(\xi_{(2,3)}=5^-)}$$

vrijednosti uzdužnih sila:

$$-T_{2,1} - \frac{3}{5}T_{2,3} - \frac{4}{5}N_{2,3} = 0 \qquad \text{(ravnoteža projekcija sila u čvoru 2 na os } z\text{)}$$

$$N_{2,3} = \frac{5}{4}\left(-T_{2,1} - \frac{3}{5}T_{2,3}\right) = -0{,}000\,147 \simeq 0 = N_{(\xi_{(2,3)}=0^+)}$$

$$N_{2,3} + N_{3,2} = 0 \qquad \text{(ravnoteža sila usporednih s osi štapa } \{2,3\}\text{)}$$

$$N_{3,2} = -N_{2,3} \simeq 0 = N_{(\xi_{(2,3)}=5^-)}$$

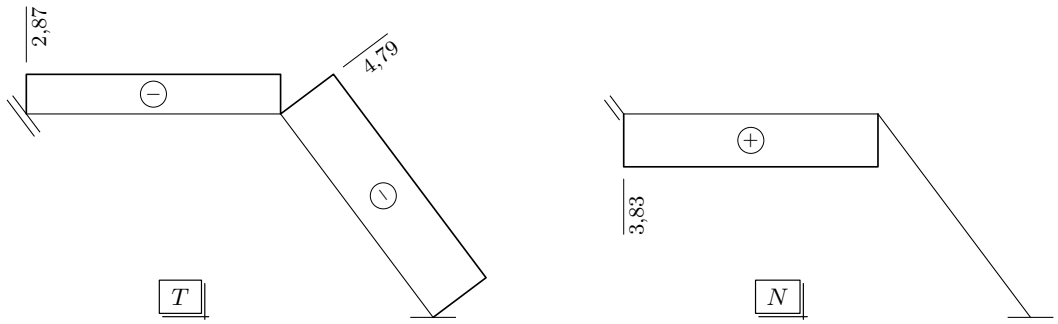
$$-N_{2,1} + \frac{4}{5}T_{2,3} - \frac{3}{5}N_{2,3} = 0 \qquad \text{(ravnoteža projekcija sila u čvoru 2 na os } x\text{)}$$

$$N_{2,1} = \frac{4}{5}T_{2,3} = 3{,}828\,131\,\text{kN} = N_{(\xi_{(2,1)}=5^-)}$$

$$N_{1,2} + N_{2,1} = 0 \qquad \text{(ravnoteža sila usporednih s osi štapa } \{1,2\}\text{)}$$

$$N_{1,2} = -N_{2,1} = -3{,}828\,131\,\text{kN} = -N_{(\xi_{(1,2)}=0^+)}$$

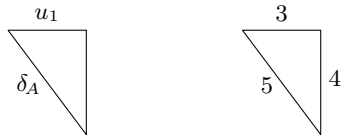
dijagrami poprečnih i uzdužnih sila:



duljina pomaka točke A:

geometrijski postupak:

nepoznanica u_1 (koja to više nije, jer smo je izračunali) duljina je horizontalne komponente pomaka točke A



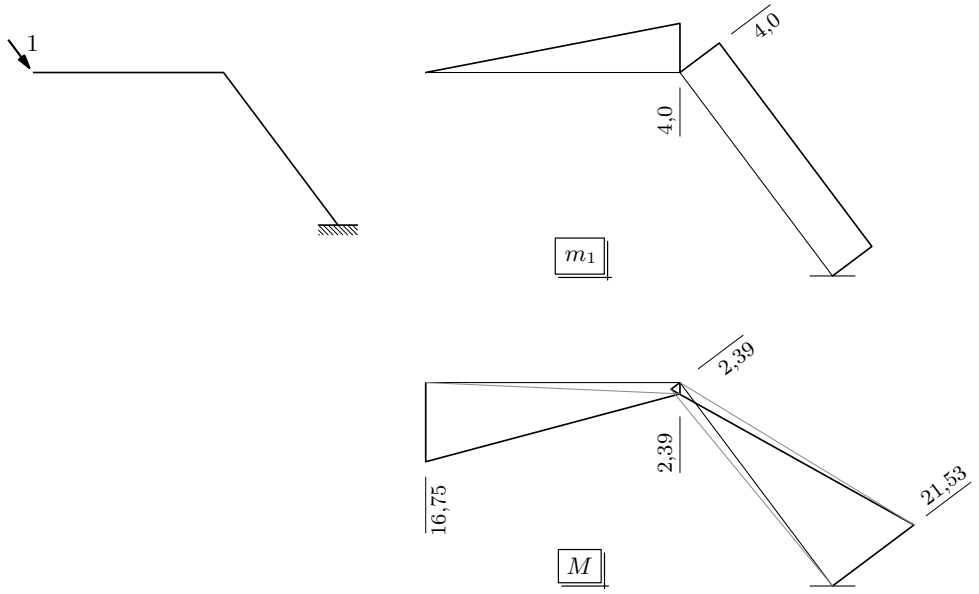
$$\frac{\delta_A}{u_1} = \frac{5}{4}$$

$$\delta_A = \frac{5}{4}u_1 = \frac{5}{4}\cdot 0{,}001\,163\,08$$

$$= 0{,}001\,938\,47\,\text{m}$$

$$= 1{,}94\,\text{mm}$$

metoda jedinične sile & redukcijski stavak:



redukcijski stavak: jedinična sila na bilo kojem osnovnom sistemu za metodu sila

[domaća zabava: objasnite crtanje dijagrama m_1!]

pomak neke točke zbroj je pomaka od utjecaja momenata savijanja i pomaka od utjecaja prisilnoga pomaka na osnovni sistem za metodu sila (ako prisilni pomak utječe na njega)

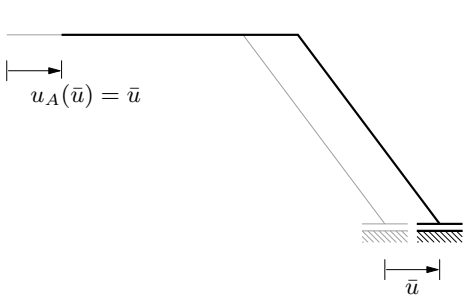
utjecaj momenata savijanja:

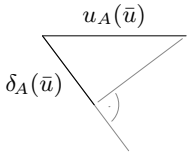
$$\delta_A(M) \;=\; \sum \int \frac{m_1\, M}{EI}\,\mathrm{d}s$$

$$= \;\frac{1}{EI}\left[\left(\frac{1}{2}\cdot 16{,}75\cdot 5\right)\left(\frac{1}{3}\cdot 4\right)(-1) \;+\; \left(\frac{1}{2}\cdot 2{,}39\cdot 5\right)\left(\frac{2}{3}\cdot 4\right)(-1)\right.$$

$$\left. +\; \left(\frac{1}{2}\cdot 2{,}39\cdot 5\right)(4)\,(-1) \;+\; \left(\frac{1}{2}\cdot 21{,}53\cdot 5\right)(4)\right] \;=\; 0{,}000\,738\,477$$

utjecaj prisilnoga pomaka na osnovni sistem za metodu sila:



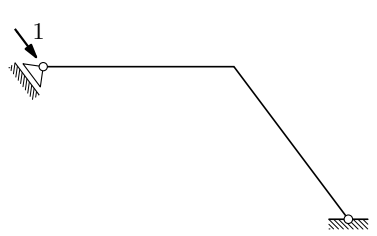


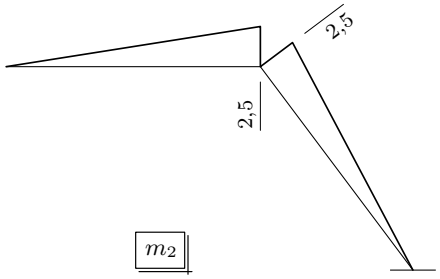
$$\frac{\delta_A(\bar{u})}{u_A(\bar{u})} \;=\; \frac{3}{5}$$

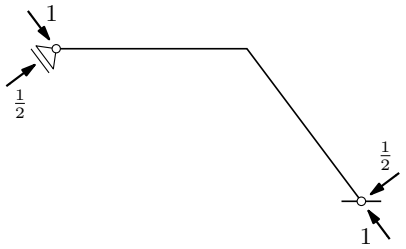
$$\delta_A(\bar{u}) \;=\; \frac{3}{5}\,u_A(\bar{u}) \;=\; \frac{3}{5}\,\bar{u} \;=\; 0{,}001\,2$$

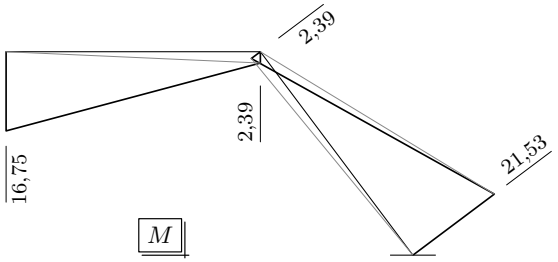
$$\delta_A \;=\; \delta_A(M) \;+\; \delta_A(\bar{u}) \;=\; 0{,}000\,738\,477 \;+\; 0{,}001\,2 \;=\; 0{,}001\,938\,48\,\mathrm{m} \;=\; 1{,}94\,\mathrm{mm}$$

ili:

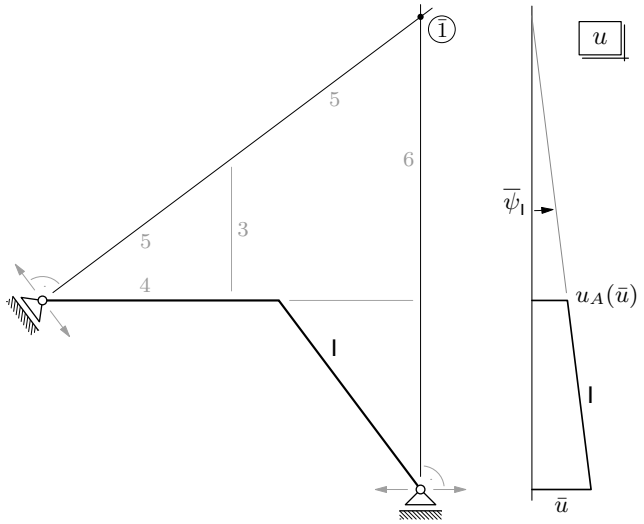


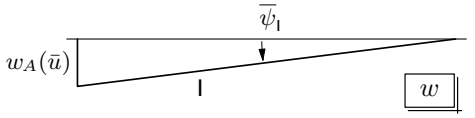






$$\delta_A(M) \;=\; \frac{1}{EI}\left[\left(\frac{1}{2}\cdot 16{,}75\cdot 5\right)\left(\frac{1}{3}\cdot 2{,}5\right)(-1) \;+\; \left(\frac{1}{2}\cdot 2{,}39\cdot 5\right)\left(\frac{2}{3}\cdot 2{,}5\right)(-1)\right.$$
$$\left. +\; \left(\frac{1}{2}\cdot 2{,}39\cdot 5\right)\left(\frac{2}{3}\cdot 2{,}5\right)(-1) \;+\; \left(\frac{1}{2}\cdot 21{,}53\cdot 5\right)\left(\frac{1}{3}\cdot 2{,}5\right)\right]$$
$$=\; -0{,}000\,061\,471\,2$$



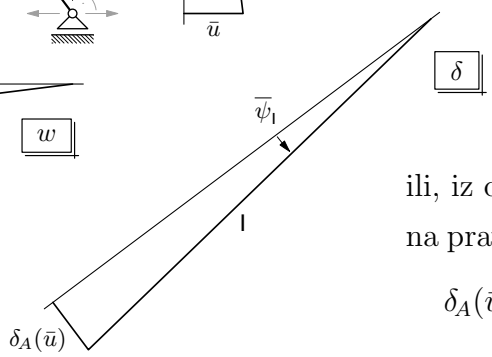


$$\overline{\psi}_{\mathsf{I}} \;=\; \frac{\bar{u}}{10}$$

$$u_A(\bar{u}) \;=\; \overline{\psi}_{\mathsf{I}}\cdot 6 \;=\; 0{,}001\,2$$

$$w_A(\bar{u}) \;=\; \overline{\psi}_{\mathsf{I}}\cdot 8 \;=\; 0{,}001\,6$$

$$\delta_A(\bar{u}) \;=\; \sqrt{u_A^2(\bar{u}) \;+\; w_A^2(\bar{u})}$$
$$=\; 0{,}002$$

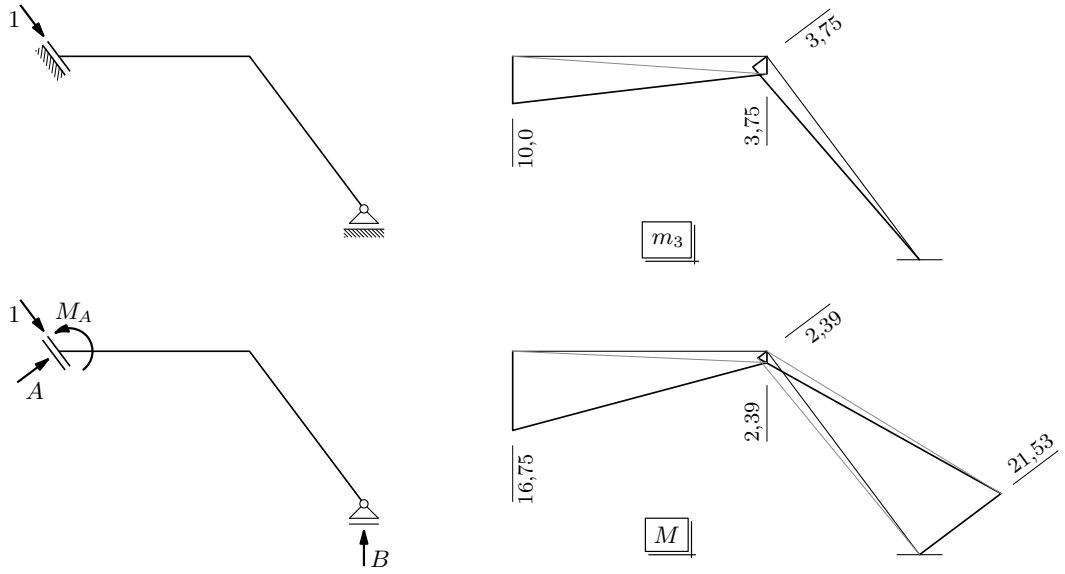


ili, iz dijagrama projekcija na pravac pomaka točke A:

$$\delta_A(\bar{u}) \;=\; \overline{\psi}_{\mathsf{I}}\cdot 10 \;=\; 0{,}002$$

$$\delta_A \;=\; \delta_A(M) \;+\; \delta_A(\bar{u}) \;=\; -0{,}000\,061\,471\,2 \;+\; 0{,}002 \;=\; 0{,}001\,938\,53\,\text{m} \;=\; 1{,}94\,\text{mm}$$

ili, osnovni sistem za metodu sila na koji prisilni pomak ne utječe:



rješavanje osnovnoga sistema (za metodu sila) na koji djeluje jedinična sila:

$$\sum F_x \;=\; 0\;: \qquad 1^h \;+\; A^h \;=\; 0$$

$$A^h \;=\; -1^h \;=\; -\frac{3}{5}$$

$$\frac{A}{A^h} \;=\; \frac{5}{4} \quad\Rightarrow\quad A \;=\; \frac{5}{4}\,A^h \;=\; -\frac{3}{4} \;=\; 0{,}75$$

$$\sum F_z \;=\; 0\;: \qquad 1^v \;-\; A^v \;-\; B \;=\; 0$$

$$\frac{A^v}{A^h} \;=\; \frac{3}{4} \quad\Rightarrow\quad A^v \;=\; \frac{3}{4}\,A^h \;=\; \frac{3}{4}\left(-\frac{3}{5}\right) \;=\; -\frac{9}{20}$$

$$B \;=\; 1^v \;-\; A^v \;=\; \frac{4}{5} \;-\; \left(-\frac{9}{20}\right) \;=\; \frac{5}{4} \;=\; 1{,}25$$

$$\sum M_{/A} \;=\; 0\;: \qquad M_A \;+\; B\cdot 8 \;=\; 0$$

$$M_A \;=\; -B\cdot 8 \;=\; -\frac{5}{4}\cdot 8 \;=\; -10$$

vrijednost momenta savijanja u spoju štapova: $\quad M_{(x=5)} \;=\; B\cdot 3 \;=\; \dfrac{15}{4} \;=\; 3{,}75$

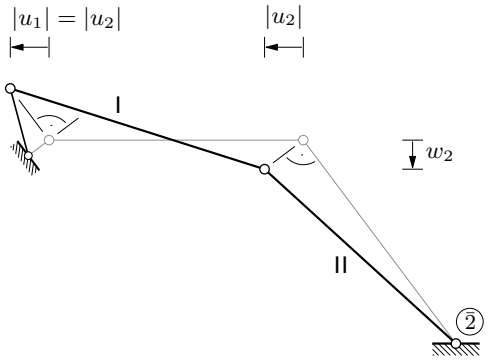
$$\begin{aligned}\delta_A \;=\; \delta_A(M) \;=\; \frac{1}{EI}\Bigg[&\left(\frac{1}{2}\cdot 16{,}75\cdot 5\right)\left(\frac{2}{3}\cdot 10{,}0 + \frac{1}{3}\cdot 3{,}75\right)\\ &+\left(\frac{1}{2}\cdot 2{,}39\cdot 5\right)\left(\frac{1}{3}\cdot 10{,}0 + \frac{2}{3}\cdot 3{,}75\right)\\ &+\left(\frac{1}{2}\cdot 2{,}39\cdot 5\right)\left(\frac{2}{3}\cdot 3{,}75\right) + \left(\frac{1}{2}\cdot 21{,}53\cdot 5\right)\left(\frac{1}{3}\cdot 3{,}75\right)(-1)\Bigg]\end{aligned}$$

$$=\; 0{,}001\,938\,40\,\text{m} \;=\; 1{,}94\,\text{mm}$$

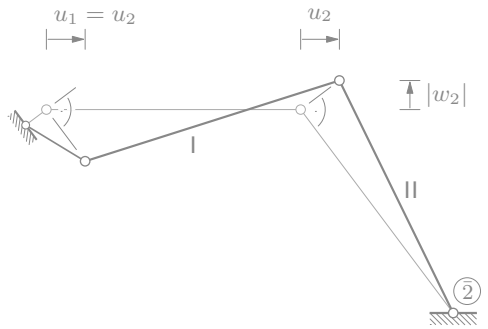
rješavanje pomoću desnoga osnovnog sistema za inženjersku metodu pomakā

(najgornja slika na stranici 2)**:**

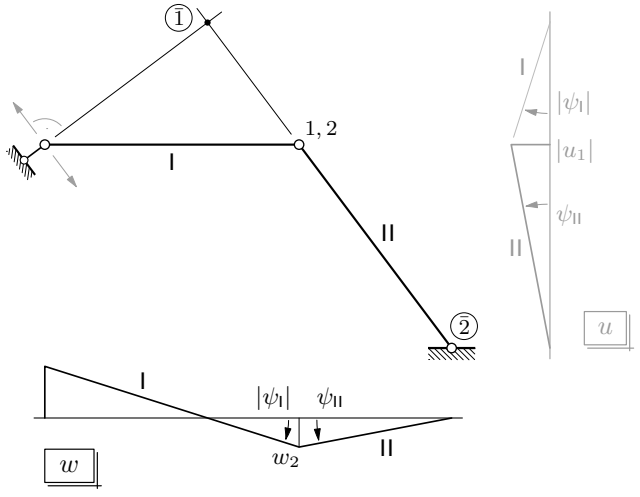
zôra radi, plan pomakā za neovisni translacijski pomak $\vec{w}_2$:

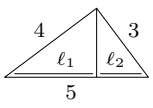


uzeli smo, kao što je uobičajeno, da je pomak $\vec{w}_2$ prema dolje; uzmemo li da je taj pomak prema gore i još da je (za crtež) $w_2 = 3\,u_2/4 = 3\,u_1/4$, plan pomakā (kao ni dijagrami projekcija pomakā) neće se razlikovati od plana pomakā (i dijagrama projekcija pomakā) za pomak $\vec{u}_1$ prikazanoga na stranici 2:



izrazi za kutove $\psi_{i,j}$ kao funkcije neovisnoga translacijskog pomaka w_2 (izvedeni pomoću dijagramā projekcijā pomakā na vertikalnu os):





$$\frac{\ell_2}{3} = \frac{3}{5}$$

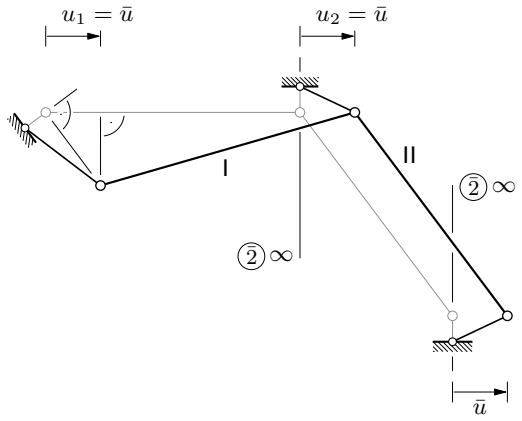
$$\ell_2 = \frac{9}{5}$$

$$\psi_\mathrm{I} = -\frac{w_2}{\ell_2} = -\frac{5}{9}\,w_2 = \psi_{\{1,2\}}$$

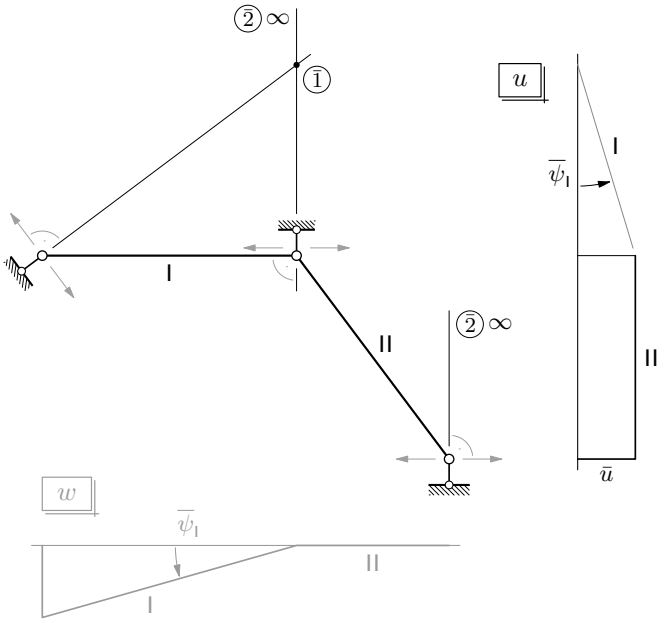
$$\psi_\mathrm{II} = \frac{w_2}{3} = \psi_{\{2,3\}}$$

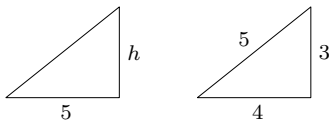
(potpunosti radi, skiciran je i dijagram projekcija pomakā na horizontalnu os)

zôra radi, plan pomakā za prisilni pomak ležaja:



izračunavanje kutova $\overline{\psi}_{i,j}$ pomoću dijagramā projekcijā pomakā na horizontalnu os:





$$\frac{h}{5} = \frac{3}{4} \quad \Rightarrow \quad h = \frac{15}{4}$$

$$\overline{\psi}_{\mathrm{I}} \;=\; \frac{\bar{u}}{h} \;=\; \frac{4}{15}\,\bar{u} \;=\; \overline{\psi}_{\{1,2\}}$$

$$\overline{\psi}_{\mathrm{II}} \;=\; 0 \;=\; \overline{\psi}_{\{2,3\}}$$

(potpunosti radi, skiciran je i dijagram projekcija pomakā na vertikalnu os)

vrijednosti momenata upetosti:

$$\overline{M}_{1,2} \;=\; \overline{M}_{2,1} \;=\; -6\,k_{\{1,2\}}\,\overline{\psi}_{\{1,2\}} \;=\; -6\;\frac{EI}{5}\;\frac{4}{15}\,\bar{u}$$

$$=\; -6\cdot\frac{162\,000}{5}\cdot\frac{4}{15}\cdot 0{,}002 \;=\; -103{,}68\,\mathrm{kNm}$$

(izrazi za) vrijednosti momenata savijanja na krajevima štapova:

$$M_{1,2} \;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} \;+\; \overline{M}_{1,2}$$

$$=\; 2\cdot 32\,400\,\varphi_2 \;-\; 6\cdot 32\,400\left(-\frac{5}{9}\,w_2\right) - 103{,}68 \;=\; 64\,800\,\varphi_2 \;+\; 108\,000\,w_2 \;-\; 103{,}68$$

$$M_{2,1} \;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} \;+\; \overline{M}_{2,1}$$

$$\;=\; 4\cdot 32\,400\,\varphi_2 \;-\; 6\cdot 32\,400\left(-\frac{5}{9}\,w_2\right) - 103{,}68 \;=\; 129\,600\,\varphi_2 \;+\; 108\,000\,w_2 \;-\; 103{,}68$$

$$M_{2,3} \;=\; 4\,k_{\{2,3\}}\,\varphi_2 \;-\; 6\,k_{\{2,3\}}\,\psi_{\{2,3\}}$$

$$\;=\; 4\cdot 32\,400\,\varphi_2 \;-\; 6\cdot 32\,400\cdot\frac{w_2}{3} \;=\; 129\,600\,\varphi_2 \;-\; 64\,800\,w_2$$

$$M_{3,2} \;=\; 2\,k_{\{2,3\}}\,\varphi_2 \;-\; 6\,k_{\{2,3\}}\,\psi_{\{2,3\}}$$

$$\;=\; 2\cdot 32\,400\,\varphi_2 \;-\; 6\cdot 32\,400\cdot\frac{w_2}{3} \;=\; 64\,800\,\varphi_2 \;-\; 64\,800\,w_2$$

jednadžba ravnoteže momenata u čvoru 2:

$$-M_{2,1} \;+\; (-M_{2,3}) \;=\; 0 \qquad \Big|\quad \times(-1)$$

$$\big[129\,600\,\varphi_2 \;+\; 108\,000\,w_2 \;-\; 103{,}68\big] \;+\; \big[129\,600\,\varphi_2 \;-\; 64\,800\,w_2\big] \;=\; 0$$

$$259\,200\,\varphi_2 \;+\; 43\,200\,w_2 \;-\; 103{,}68 \;=\; 0$$

jednadžba rada na virtualnim pomacima:

(„vodeći" je virtualni pomak pomak $\delta\vec{w}_2$ koji odgovara neovisnom translacijskom pomaku $\vec{w}_2$, pa je dijagram projekcija (virtualnih) pomaka na vertikalnu os jednak dijagramu prikazanom na najdonjoj slici na stranici 10, uz zamjene $w_2 \to \delta w_2$ i $\psi_{\mathrm{i}} \to \delta\psi_{\mathrm{i}}$, a veza je kutova $\delta\psi_{\{i,j\}}$ i orijentirane duljine δw_2 ista kao veza $\psi_{\{i,j\}}$ i w_2:

$$\delta\psi_{\{1,2\}} \;=\; {}^{-5\,\delta w_2}\!/_{9} \qquad \& \qquad \delta\psi_{\{2,3\}} \;=\; {}^{\delta w_2}\!/_{3}\;)$$

$$\big(M_{1,2} \;+\; M_{2,1}\big)\,\delta\psi_{\{1,2\}} \;+\; \big(M_{2,3} \;+\; M_{3,2}\big)\,\delta\psi_{\{2,3\}} \;=\; 0$$

$$\big(64\,800\,\varphi_2 \;+\; 108\,000\,w_2 \;-\; 103{,}68 \;+\; 129\,600\,\varphi_2 \;+\; 108\,000\,w_2 \;-\; 103{,}68\big)\cdot\left(-\frac{5}{9}\,\delta w_2\right)$$

$$+\; \big(129\,600\,\varphi_2 \;-\; 64\,800\,w_2 \;+\; 64\,800\,\varphi_2 \;-\; 64\,800\,w_2\big)\cdot\frac{\delta w_2}{3} \;=\; 0$$

$$\big(-43\,200\,\varphi_2 \;-\; 162\,667\,w_2 \;+\; 115{,}2\big)\,\delta w_2 \;=\; 0 \quad \forall\,\delta w_2$$

$$-43\,200\,\varphi_2 \;-\; 162\,667\,w_2 \;+\; 115{,}2 \;=\; 0 \qquad \Big|\quad \times(-1)$$

$$43\,200\,\varphi_2 \;+\; 162\,667\,w_2 \;-\; 115{,}2 \;=\; 0$$

sustav jednadžbi i njegovo rješenje:

$$\begin{aligned} 259\,200\,\varphi_2 \;+\;\; 43\,200\,w_2 \;&=\; 103{,}68 \\ 43\,200\,\varphi_2 \;+\; 162\,667\,w_2 \;&=\; 115{,}2 \end{aligned} \qquad \text{(sustav je simetričan)}$$

$$\varphi_2 \;=\; 0{,}000\,295\,026 \qquad \& \qquad w_2 \;=\; 0{,}000\,629\,844\ \mathrm{m}$$

vrijednosti momenata savijanja na krajevima štapova:

$$M_{1,2} = 64\,800 \cdot 0{,}000\,295\,026 + 108\,000 \cdot 0{,}000\,629\,844 - 103{,}68 = -16{,}775\,2\ \mathrm{kNm}$$

$$M_{2,1} = 129\,600 \cdot 0{,}000\,295\,026 + 108\,000 \cdot 0{,}000\,629\,844 - 103{,}68 = 2{,}578\,52\ \mathrm{kNm}$$

$$M_{2,3} = 129\,600 \cdot 0{,}000\,295\,026 - 64\,800 \cdot 0{,}000\,629\,844 = -2{,}578\,52\ \mathrm{kNm}$$

$$M_{3,2} = 64\,800 \cdot 0{,}000\,295\,026 - 64\,800 \cdot 0{,}000\,629\,844 = -21{,}696\,2\ \mathrm{kNm}$$

(Nebitne razlike (iza decimalnoga zareza) između ovih vrijednosti i vrijednostī dobivenih rješavanjem pomoću lijevoga osnovnog sistema posljedice su zanemarivanja pogrešaka zaokruživanja (na šest značajnih znamenaka) u pojedinim koracima proračuna i njihova „gomilanja”.)

nastavak je priče (dijagrami *&* duljina pomaka) isti kao za lijevi osnovni sistem
(uz lagana prilagođavanja vrijednostī)