GS 2. — popravni kolokvij (2023./2024.)

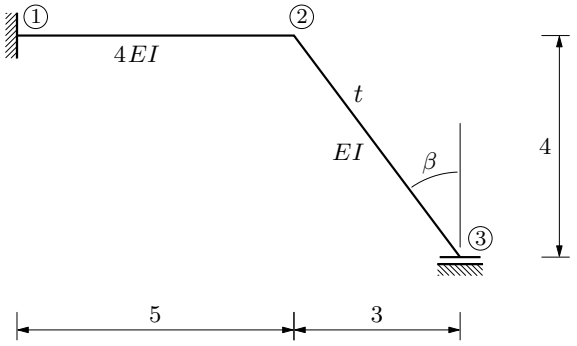
Zadatak 1.

Inženjerskom metodom pomakā nacrtajte dijagram momenata savijanja!

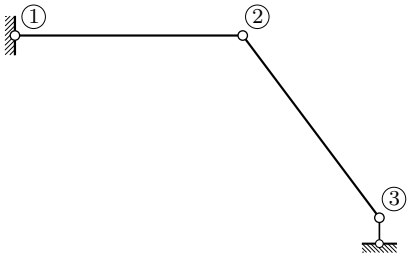
$t = 20^\circ\,\mathrm{C}$ (u štapu 2–3)

$EI = 20250\,\mathrm{kNm}^2$

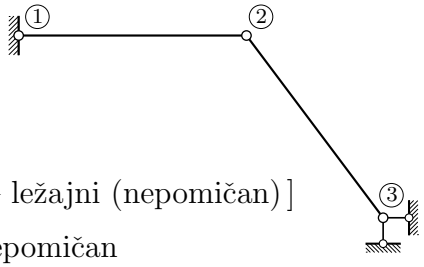
$\alpha_t = 1 \cdot 10^{-5}\,\mathrm{K}^{-1}$



zglobna shema i kinematička analiza zglobne sheme (dodavanje spoj(ev)a s podlogom):



$s_{\min} = 2 \cdot 2 - 3 = 1$

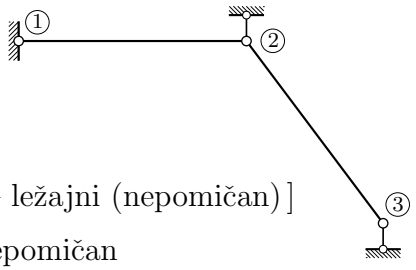


[čvor 1 – ležajni (nepomičan)]

čvor 3 nepomičan

$\rightarrow$ čvor 2 nepomičan

ili



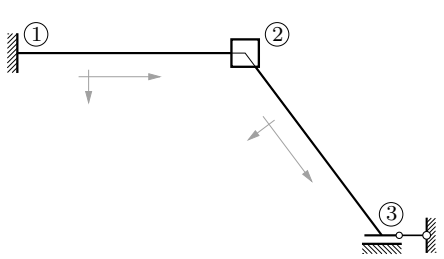
[čvor 1 – ležajni (nepomičan)]

čvor 2 nepomičan

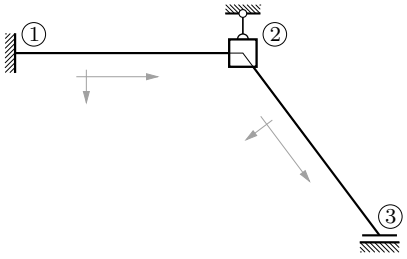
$\rightarrow$ čvor 3 nepomičan

$\Rightarrow \quad s = s_{\min} = 1$

osnovni sistem za inženjersku metodu pomakā:

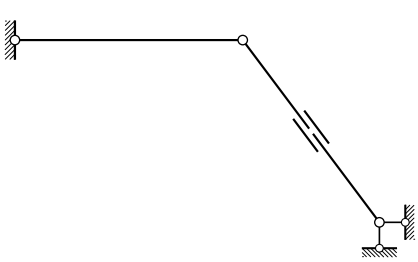


ili

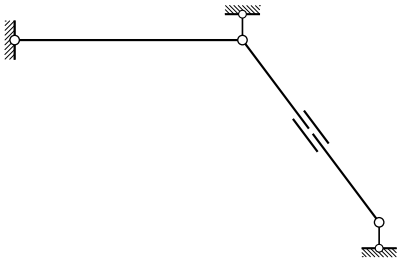


nepoznanice: φ_2 i u_3 nepoznanice: φ_2 i w_2

mehanizam za utjecaj promjene temperature:



ili

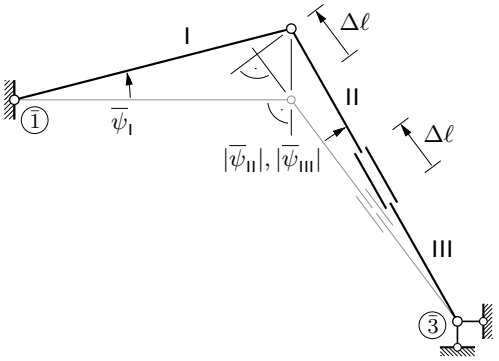


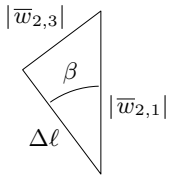
produljenje štapa 2–3 zbog jednolike promjene temperature:

$$\Delta\ell \;=\; \alpha_t\, t\, \ell_{\{2,3\}} \;=\; 1\cdot 10^{-5}\cdot 20\cdot 5 \;=\; 0{,}001$$

rješavanje pomoću lijevoga osnovnog sistema:

izračunavanje kutova $\overline{\psi}_{\{i,j\}}$ pomoću plana pomakā za utjecaj promjene temperature:





(u čvoru 2)

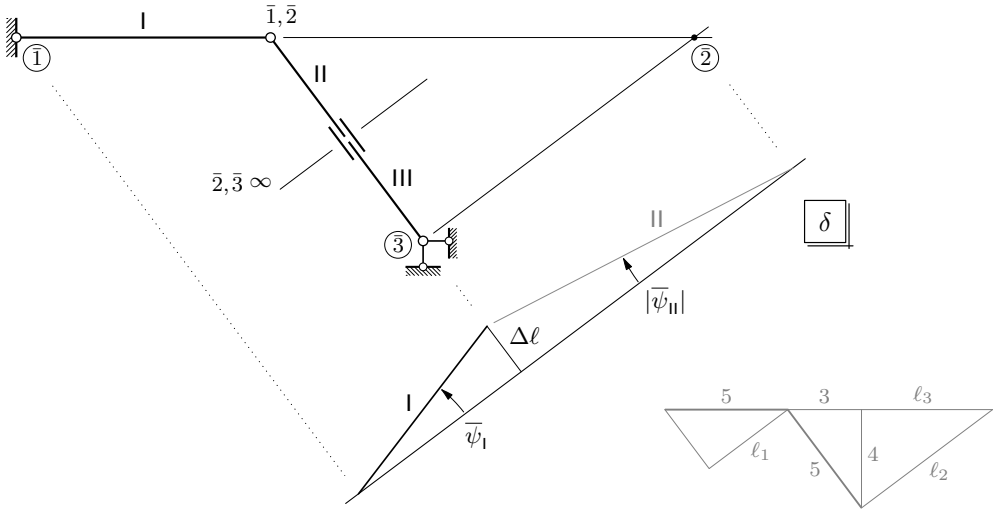
$$\frac{|\overline{w}_{2,3}|}{\Delta\ell} \;=\; \frac{3}{4} \qquad \Rightarrow \qquad |\overline{w}_{2,3}| \;=\; \frac{3}{4}\,\Delta\ell \;=\; 0{,}000\,75$$

$$\frac{\Delta\ell}{|\overline{w}_{2,1}|} \;=\; \frac{4}{5} \qquad \Rightarrow \qquad |\overline{w}_{2,1}| \;=\; \frac{5}{4}\,\Delta\ell \;=\; 0{,}001\,25$$

$$\overline{\psi}_{\mathrm{I}} \;=\; \frac{|\overline{w}_{2,1}|}{\ell_{\{1,2\}}} \;=\; 0{,}000\,25 \;=\; \overline{\psi}_{\{1,2\}}$$

$$\overline{\psi}_{\mathrm{II}} \;=\; \overline{\psi}_{\mathrm{III}} \;=\; -\frac{|\overline{w}_{2,3}|}{\ell_{\{2,3\}}} \;=\; -0{,}000\,15 \;=\; \overline{\psi}_{\{2,3\}}$$

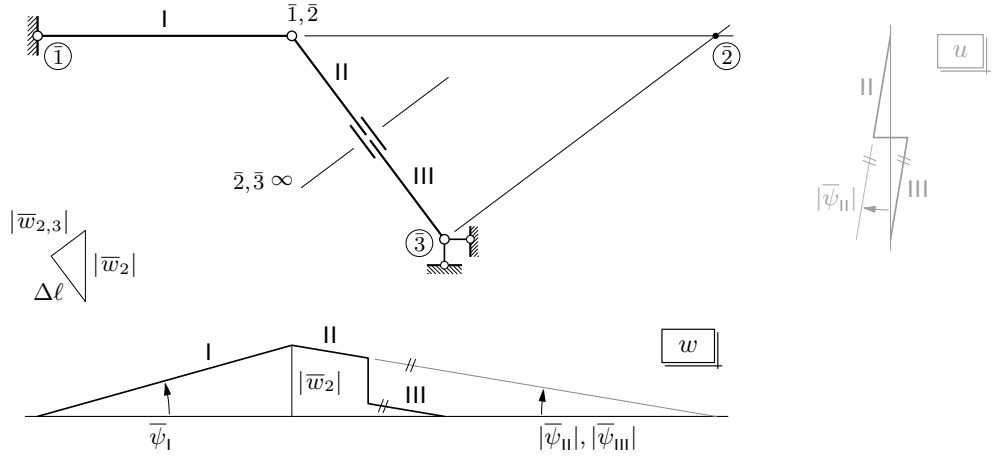
ili izračunavanje kutova $\overline{\psi}_{\{i,j\}}$ pomoću dijagrama projekcija pomakā na os štapa 2–3:



$$\overline{\psi}_{\mathrm{I}} \;=\; \frac{\Delta\ell}{\ell_1} \;=\; 0{,}000\,25 \;=\; \overline{\psi}_{\{1,2\}} \qquad \frac{\ell_1}{5} \;=\; \frac{4}{5} \qquad \Rightarrow \qquad \ell_1 \;=\; 4$$

$$\overline{\psi}_{\mathrm{II}} \;=\; -\frac{\Delta\ell}{\ell_2} \;=\; -0{,}000\,15 \;=\; \overline{\psi}_{\{2,3\}} \qquad \frac{\ell_2}{4} \;=\; \frac{5}{3} \qquad \Rightarrow \qquad \ell_2 \;=\; \frac{20}{3}$$

ili izračunavanje kutova $\overline{\psi}_{\{i,j\}}$ pomoću dijagrama projekcija pomakā na vertikalnu os:



(Treba ovdje naglasiti (prije je, pri crtanju plana pomakā bilo (ili barem trebalo biti) očito) da $\overrightarrow{\Delta\ell}$ (vektor duljine $\Delta\ell$ na osi štapa 2–3) nije pomak $\vec{\underline{\delta}}_2$ čvora 2, tako da ni projekcije vektora $\overrightarrow{\Delta\ell}$ na vertikalnu i horizontalnu os nisu projekcije pomaka $\vec{\underline{\delta}}_2$ na te osi. Pomak $\overrightarrow{\Delta\ell}$ samo je komponenta pomaka $\vec{\underline{\delta}}_2$ na osi štapa 2–3, uzrokovana njegovim produljenjem. Štap 2–3 se zbog spoja sa štapom 1–2 i zaokreće (oko pola $\bar{2}$), pa $\vec{\underline{\delta}}_2$ ima i komponentu $\vec{\underline{w}}_{2,3}$ okomitu na os štapa 2–3. Kako čvor 2 „pripada" i štapu 1–2, pomiče se po okomici na os toga štapa, tako da zbroj komponenata $\overrightarrow{\Delta\ell}$ i $\vec{\underline{w}}_{2,3}$ pomaka $\vec{\underline{\delta}}_2$ mora biti na toj okomici (pogledajte i plan pomakā na najdonjoj slici na stranici 2), pa se $\vec{\underline{\delta}}_2$ i njegova projekcija $\vec{\underline{w}}_2$ na vertikalnu os (duljine $\overline{w}_2$) poklapaju, $\vec{\underline{\delta}}_2 = \vec{\underline{w}}_2$, a projekcija $\vec{\underline{u}}_2$ na horizontalnu os iščezava, $\vec{\underline{u}}_2 = \vec{0}$, $\bar{u}_2 = 0$.)

$$\frac{\Delta\ell}{|\overline{w}_2|} = \frac{4}{5} \qquad \Rightarrow \qquad |\overline{w}_2| = \frac{5}{4}\,\Delta\ell = 0{,}001\,25$$

$$\overline{\psi}_{\mathrm{I}} = \frac{|\overline{w}_2|}{\ell_{\{1,2\}}} = 0{,}000\,25 = \overline{\psi}_{\{1,2\}}$$

$$\overline{\psi}_{\mathrm{II}} = \overline{\psi}_{\mathrm{III}} = -\frac{|\overline{w}_2|}{3+\ell_3} = -0{,}000\,15 = \overline{\psi}_{\{2,3\}}$$

$$\frac{\ell_3}{4} = \frac{4}{3} \qquad \Rightarrow \qquad \ell_3 = \frac{16}{3}$$

(sivi dio donje slike na prethodnoj stranici)

(Dijagram projekcija pomakā na horizontalnu os nacrtan je potpunosti radi, iako je nepotreban i, u stvari, neupotrebljiv — pomoću njega se $\overline{\psi}_{\mathrm{I}}$ ne može odrediti.)

vrijednosti momenata upetosti:

$$\overline{M}_{1,2} = \overline{M}_{2,1} = -6\,k_{\{1,2\}}\,\overline{\psi}_{\{1,2\}} = -6\cdot 16\,200\cdot 0{,}000\,25 = -24{,}3\,\mathrm{kNm}$$

$$k_{\{1,2\}} = \frac{4EI}{\ell_{\{1,2\}}} = 16\,200\mathrm{kNm}$$

$$\overline{M}_{2,3} = \overline{M}_{3,2} = -6\,k_{\{2,3\}}\,\overline{\psi}_{\{2,3\}} = -6\cdot 4\,050\cdot(-0{,}000\,15) = 3{,}645\,\mathrm{kNm}$$

$$k_{\{2,3\}} = \frac{EI}{\ell_{\{2,3\}}} = 4\,050\mathrm{kNm}$$

vrijednosti momenata na krajevima štapova:

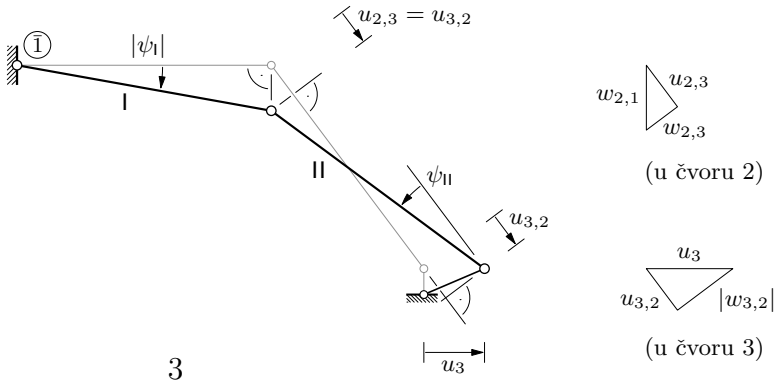
$$M_{1,2} = 2\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} + \overline{M}_{1,2}$$

$$M_{2,1} = 4\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} + \overline{M}_{2,1}$$

$$M_{2,3} = 4\,k_{\{2,3\}}\,\varphi_2 - 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} + \overline{M}_{2,3}$$

$$M_{3,2} = 2\,k_{\{2,3\}}\,\varphi_2 - 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} + \overline{M}_{2,3}$$

izračunavanje kutova $\psi_{i,j}$ pomoću plana pomakā za neovisni translacijski pomak $\vec{u}_3$ (zapetljano & ružno):



u čvoru 3:

$$\frac{u_{3,2}}{u_3} = \frac{3}{5} \qquad \Rightarrow \qquad u_{3,2} = \frac{3}{5}\,u_3$$

$$\frac{|w_{3,2}|}{u_3} = \frac{4}{5} \qquad \Rightarrow \qquad |w_{3,2}| = \frac{4}{5}\,u_3$$

u čvoru 2:

$$u_{2,3} = u_{3,2}$$

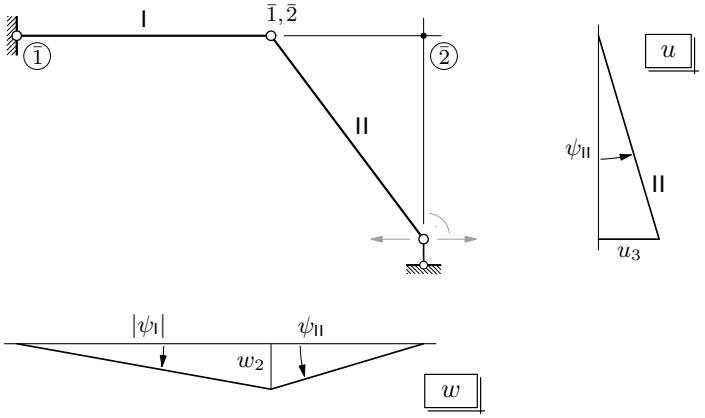
$$\frac{w_{2,3}}{u_{2,3}} = \frac{3}{4} \qquad \Rightarrow \qquad w_{2,3} = \frac{3}{4}\,u_{2,3} = \frac{9}{20}\,u_3$$

$$\frac{w_{2,1}}{u_{2,3}} = \frac{5}{4} \qquad \Rightarrow \qquad w_{2,1} = \frac{5}{4}\,u_{2,3} = \frac{3}{4}\,u_3$$

$$\psi_{\{1,2\}} = \psi_{\mathrm{I}} = -\frac{w_{2,1}}{\ell_{\{1,2\}}} = -\frac{3}{20}\,u_3$$

$$\psi_{\{2,3\}} = \psi_{\mathrm{II}} = \frac{w_{2,3} + |w_{3,2}|}{\ell_{\{2,3\}}} = \frac{1}{4}\,u_3$$

ili izračunavanje kutova $\psi_{i,j}$ pomoću dijagramā projekcijā pomakā na horizontalnu i na vertikalnu os:



$$\psi_{\{2,3\}} \;=\; \psi_{\text{II}} \;=\; \frac{u_3}{4} \;=\; 0{,}25\,u_3$$

$$w_2 \;=\; \psi_{\text{II}} \cdot 3 \;=\; \frac{3}{4}\,u_3$$

$$\psi_{\{1,2\}} \;=\; \psi_{\text{I}} \;=\; -\frac{w_2}{5} \;=\; -\frac{3}{20}\,u_3 \;=\; -0{,}15\,u_3$$

vrijednosti momenata na krajevima štapova (još jednom):

$$\begin{aligned} M_{1,2} \;&=\; 2\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} \;+\; \overline{M}_{1,2} \\ &=\; 2\cdot 16\,200\,\varphi_2 \;-\; 6\cdot 16\,200\cdot(-0{,}15\,u_3) \;+\; (-24{,}3) \\ &=\; 32\,400\,\varphi_2 \;+\; 14\,580\,u_3 \;-\; 24{,}3 \end{aligned}$$

$$\begin{aligned} M_{2,1} \;&=\; 4\,k_{\{1,2\}}\,\varphi_2 \;-\; 6\,k_{\{1,2\}}\,\psi_{\{1,2\}} \;+\; \overline{M}_{2,1} \\ &=\; 4\cdot 16\,200\,\varphi_2 \;-\; 6\cdot 16\,200\cdot(-0{,}15\,u_3) \;+\; (-24{,}3) \\ &=\; 64\,800\,\varphi_2 \;+\; 14\,580\,u_3 \;-\; 24{,}3 \end{aligned}$$

$$\begin{aligned} M_{2,3} \;&=\; 4\,k_{\{2,3\}}\,\varphi_2 \;-\; 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} \;+\; \overline{M}_{2,3} \\ &=\; 4\cdot 4\,050\,\varphi_2 \;-\; 6\cdot 4\,050\cdot 0{,}25\,u_3 \;+\; 3{,}645 \\ &=\; 16\,200\,\varphi_2 \;-\; 6\,075\,u_3 \;+\; 3{,}645 \end{aligned}$$

$$\begin{aligned} M_{3,2} \;&=\; 2\,k_{\{2,3\}}\,\varphi_2 \;-\; 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} \;+\; \overline{M}_{2,3} \\ &=\; 2\cdot 4\,050\,\varphi_2 \;-\; 6\cdot 4\,050\cdot 0{,}25\,u_3 \;+\; 3{,}645 \\ &=\; 8\,100\,\varphi_2 \;-\; 6\,075\,u_3 \;+\; 3{,}645 \end{aligned}$$

jednadžba ravnoteže momenata u čvoru 2:

$$-M_{2,1} \;+\; (-M_{2,3}) \;=\; 0 \qquad \Big|\quad \times(-1)$$

$$\big[64\,800\,\varphi_2 \;+\; 14\,580\,u_3 \;-\; 24{,}3\big] \;+\; \big[16\,200\,\varphi_2 \;-\; 6\,075\,u_3 \;+\; 3{,}645\big] \;=\; 0$$

$$81\,000\,\varphi_2 \;+\; 8\,505\,u_3 \;-\; 20{,}655 \;=\; 0$$

jednadžba rada na virtualnim pomacima:

(„vodeći” je virtualni pomak pomak $\delta\vec{u}_3$ koji odgovara neovisnom translacijskom pomaku $\vec{u}_3$, pa su plan (virtualnih) pomaka i dijagrami projekcija (virtualnih) pomaka na koordinatne osi jednaki planu i dijagramima prikazanima na prethodnoj stranici, uz zamjene $u_3 \to \delta u_3$ i $\psi_{\text{I}} \to \delta\psi_{\text{I}}$, a veza je kutova $\delta\psi_{\{i,j\}}$ i orijentirane duljine δu_3 ista kao veza $\psi_{\{i,j\}}$ i u_3:

$$\delta\psi_{\{1,2\}} \;=\; -\frac{3}{20}\,\delta u_3 \;=\; -0{,}15\,\delta u_3 \qquad \& \qquad \delta\psi_{\{2,3\}} \;=\; \frac{\delta u_3}{4} \;=\; 0{,}25\,\delta u_3\,\big)$$

$$\big(M_{1,2} \,+\, M_{2,1}\big)\,\delta\psi_{\{1,2\}} \,+\, \big(M_{2,3} \,+\, M_{3,2}\big)\,\delta\psi_{\{2,3\}} \;=\; 0$$

$$\big(32\,400\,\varphi_2 \,+\, 14\,580\,u_3 \,-\, 24{,}3 \,+\, 64\,800\,\varphi_2 \,+\, 14\,580\,u_3 \,-\, 24{,}3\big)\cdot(-0{,}15\,\delta u_3)$$
$$+\,\big(16\,200\,\varphi_2 \,-\, 6\,075\,u_3 \,+\, 3{,}645 \,+\, 8\,100\,\varphi_2 \,-\, 6\,075\,u_3 \,+\, 3{,}645\big)\,\cdot\,0{,}25\,\delta u_3 \;=\; 0$$

$$\big(-8\,505\,\varphi_2 \,-\, 7\,411{,}5\,u_3 \,+\, 9{,}102\,5\big)\,\delta u_3 \;=\; 0 \quad \forall\,\delta u_3$$

$$-8\,505\,\varphi_2 \,-\, 7\,411{,}5\,u_3 \,+\, 9{,}102\,5 \;=\; 0 \quad \Big| \quad \times(-1)$$

$$8\,505\,\varphi_2 \,+\, 7\,411{,}5\,u_3 \,-\, 9{,}102\,5 \;=\; 0$$

sustav jednadžbi i njegovo rješenje:

$$81\,000\,\varphi_2 \,+\, 8\,505 \quad u_3 \;=\; 20{,}655$$

$$8\,505\,\varphi_2 \,+\, 7\,411{,}5\,u_3 \;=\; 9{,}102\,5 \qquad \text{(sustav je simetričan)}$$

$$\varphi_2 \;=\; 0{,}000\,143\,3 \qquad \&\qquad u_3 \;=\; 0{,}001\,063\,7\ \text{m}$$

vrijednosti momenata na krajevima štapova (treći (i posljednji) put):

$$M_{1,2} \;=\; 32\,400\cdot 0{,}000\,143\,3 \,+\, 14\,580\cdot 0{,}001\,063\,7 \,-\, 24{,}3 \;=\; -4{,}148\ \text{kNm}$$

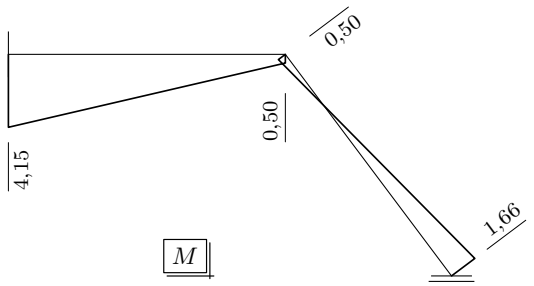
$$M_{2,1} \;=\; 64\,800\cdot 0{,}000\,143\,3 \,+\, 14\,580\cdot 0{,}001\,063\,7 \,-\, 24{,}3 \;=\; 0{,}495\ \text{kNm}$$

$$M_{2,3} \;=\; 16\,200\cdot 0{,}000\,143\,3 \,-\, 6\,075\cdot 0{,}001\,063\,7 \,+\, 3{,}645 \;=\; -0{,}496\ \text{kNm}$$

$$M_{3,2} \;=\; 8\,100\cdot 0{,}000\,143\,3 \,-\, 6\,075\cdot 0{,}001\,063\,7 \,+\, 3{,}645 \;=\; -1{,}656\ \text{kNm}$$

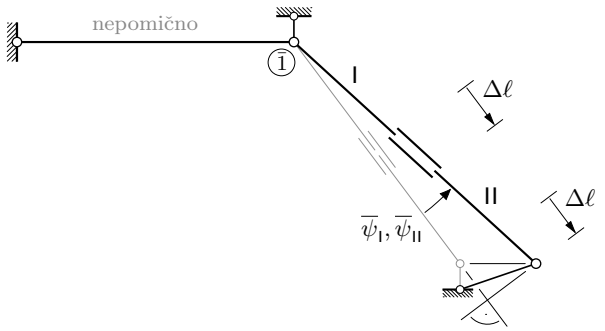
(„Neravnoteža” momenata u čvoru 2 ($M_{2,1} \neq |M_{2,3}|$) posljedica je pogrešaka zaokruživanja u međukoracima proračuna i njihova „gomilanja”.)

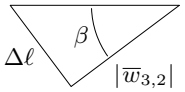
dijagram momenata savijanja:



rješavanje pomoću desnoga osnovnog sistema:

izračunavanje kuta $\overline{\psi}_{\{2,3\}}$ pomoću plana pomakā za utjecaj promjene temperature:



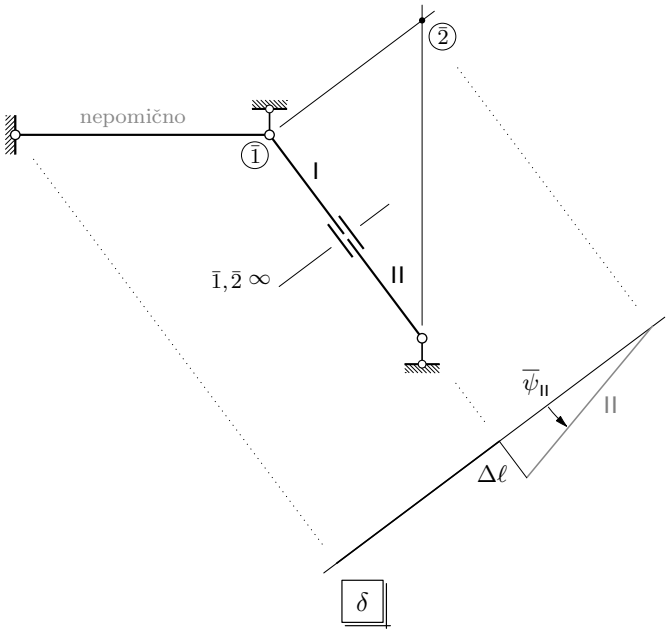


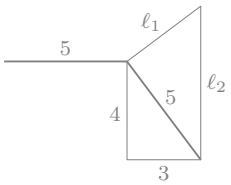
(u čvoru 3)

$$\frac{|\overline{w}_{3,2}|}{\Delta\ell} = \frac{4}{3} \qquad \Rightarrow \qquad |\overline{w}_{3,2}| = \frac{4}{3}\,\Delta\ell = 0{,}001\,333$$

$$\overline{\psi}_{\mathrm{I}} = \overline{\psi}_{\mathrm{II}} = \frac{|\overline{w}_{3,2}|}{\ell_{\{2,3\}}} = 0{,}000\,267 = \overline{\psi}_{\{2,3\}}$$

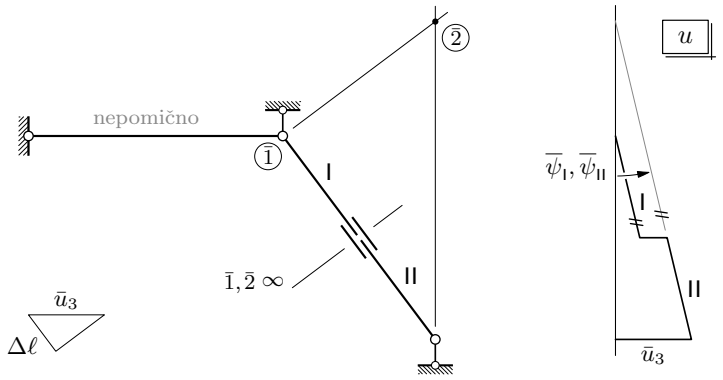
ili izračunavanje kuta $\overline{\psi}_{\{2,3\}}$ pomoću dijagrama projekcija pomakā na os štapa 2–3:





$$\overline{\psi}_{\mathrm{II}} = \frac{\Delta\ell}{\ell_1} = 0{,}000\,267 = \overline{\psi}_{\{2,3\}} \qquad \frac{\ell_1}{5} = \frac{3}{4} \qquad \Rightarrow \qquad \ell_1 = \frac{15}{4}$$

ili izračunavanje kuta $\overline{\psi}_{\{2,3\}}$ pomoću dijagrama projekcija pomakā na horizontalnu os:



$$\frac{\bar{u}_3}{\Delta\ell} = \frac{5}{3} \quad \Rightarrow \quad \bar{u}_3 = \frac{5}{3}\,\Delta\ell = 0{,}001\,667$$

$$\overline{\psi}_{\text{I}} = \overline{\psi}_{\text{II}} = \frac{\bar{u}_3}{\ell_2} = 0{,}000\,267 = \overline{\psi}_{\{2,3\}}$$

$$\frac{\ell_2}{5} = \frac{5}{4} \quad \Rightarrow \quad \ell_2 = \frac{25}{4}$$

(sivi dio donje slike na prethodnoj stranici)

vrijednosti momenata upetosti:

$$\overline{M}_{2,3} = \overline{M}_{3,2} = -6\,k_{\{2,3\}}\,\overline{\psi}_{\{2,3\}} = -6 \cdot 4\,050 \cdot 0{,}000\,267 = -6{,}488\,1\ \text{kNm}$$

vrijednosti momenata na krajevima štapova:

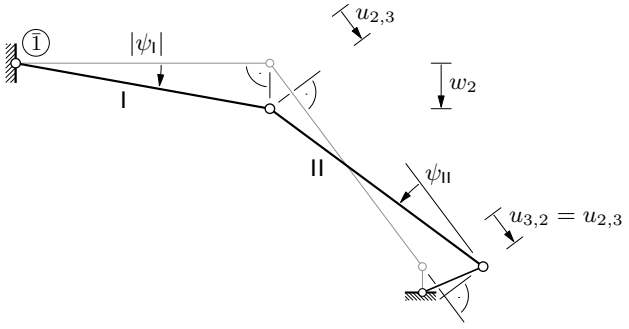
$$M_{1,2} = 2\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}$$

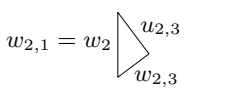
$$M_{2,1} = 4\,k_{\{1,2\}}\,\varphi_2 - 6\,k_{\{1,2\}}\,\psi_{\{1,2\}}$$

$$M_{2,3} = 4\,k_{\{2,3\}}\,\varphi_2 - 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} + \overline{M}_{2,3}$$

$$M_{3,2} = 2\,k_{\{2,3\}}\,\varphi_2 - 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} + \overline{M}_{2,3}$$

izračunavanje kutova $\psi_{i,j}$ pomoću plana pomakā za neovisni translacijski pomak $\vec{w}_2$:
(zapetljano & ružno):





(u čvoru 2)

(u čvoru 3)

(Plan je pomakā, dakako, jednak onom na gornjoj slici na stranici 5, ali s drugim „vodećim" pomakom.)

u čvoru 2:

$$w_{2,1} \;=\; w_2$$

$$\frac{u_{2,3}}{w_2} \;=\; \frac{4}{5} \qquad \Rightarrow \qquad u_{2,3} \;=\; \frac{4}{5}\, w_2$$

$$\frac{w_{2,3}}{w_2} \;=\; \frac{3}{5} \qquad \Rightarrow \qquad w_{2,3} \;=\; \frac{3}{5}\, w_2$$

u čvoru 3:

$$u_{3,2} \;=\; u_{2,3} \;=\; \frac{4}{5}\, w_2$$

$$\frac{|w_{3,2}|}{u_{3,2}} \;=\; \frac{4}{3} \qquad \Rightarrow \qquad |w_{3,2}| \;=\; \frac{4}{3}\, u_{3,2} \;=\; \frac{16}{15}\, w_2$$

$$\psi_{\{1,2\}} \;=\; \psi_{\mathrm{I}} \;=\; -\frac{w_2}{\ell_{\{1,2\}}} \;=\; -\frac{w_2}{5}$$

$$\psi_{\{2,3\}} \;=\; \psi_{\mathrm{II}} \;=\; \frac{w_{2,3} + |w_{3,2}|}{\ell_{\{2,3\}}} \;=\; \frac{w_2}{3}$$

ili izračunavanje kutova $\psi_{i,j}$ pomoću dijagrama projekcijā pomakā na vertikalnu os

dijagram je prikazan na donjoj slici na stranici 5

(na istoj slici prikazani dijagram projekcija pomakā na horizontalnu os sada nije potreban)

$$\psi_{\{1,2\}} \;=\; \psi_{\mathrm{I}} \;=\; -\frac{w_2}{5} \;=\; -0{,}2\, w_2$$

$$\psi_{\{2,3\}} \;=\; \psi_{\mathrm{II}} \;=\; \frac{w_2}{3} \;=\; 0{,}333\,3\, w_2$$

vrijednosti momenata na krajevima štapova (još jednom):

$$\begin{aligned} M_{1,2} \;&=\; 2\, k_{\{1,2\}}\, \varphi_2 \;-\; 6\, k_{\{1,2\}}\, \psi_{\{1,2\}} \\ &=\; 2 \cdot 16\,200\, \varphi_2 \;-\; 6 \cdot 16\,200 \cdot (-0{,}2\, w_2) \\ &=\; 32\,400\, \varphi_2 \;+\; 19\,440\, w_2 \end{aligned}$$

$$\begin{aligned} M_{2,1} \;&=\; 4\, k_{\{1,2\}}\, \varphi_2 \;-\; 6\, k_{\{1,2\}}\, \psi_{\{1,2\}} \\ &=\; 4 \cdot 16\,200\, \varphi_2 \;-\; 6 \cdot 16\,200 \cdot (-0{,}2\, w_2) \\ &=\; 64\,800\, \varphi_2 \;+\; 19\,440\, w_2 \end{aligned}$$

$$
\begin{aligned}
M_{2,3} &= 4\,k_{\{2,3\}}\,\varphi_2 \,-\, 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} \,+\, \overline{M}_{2,3} \\
&= 4 \cdot 4\,050\,\varphi_2 \,-\, 6 \cdot 4\,050 \cdot 0{,}333\,3\,w_2 \,-\, 6{,}488\,1 \\
&= 16\,200\,\varphi_2 \,-\, 8\,100\,w_2 \,-\, 6{,}488\,1 \\
M_{3,2} &= 2\,k_{\{2,3\}}\,\varphi_2 \,-\, 6\,k_{\{2,3\}}\,\psi_{\{2,3\}} \,+\, \overline{M}_{2,3} \\
&= 2 \cdot 4\,050\,\varphi_2 \,-\, 6 \cdot 4\,050 \cdot 0{,}333\,3\,w_2 \,-\, 6{,}488\,1 \\
&= 8\,100\,\varphi_2 \,-\, 8\,100\,w_2 \,-\, 6{,}488\,1
\end{aligned}
$$

jednadžba ravnoteže momenata u čvoru 2:

$$
\begin{aligned}
&-M_{2,1} \,+\, (-M_{2,3}) \,=\, 0 \quad \Big| \quad \times (-1) \\
&\big[64\,800\,\varphi_2 \,+\, 19\,440\,w_2\big] \,+\, \big[16\,200\,\varphi_2 \,-\, 8\,100\,w_2 \,-\, 6{,}488\,1\big] \,=\, 0 \\
&81\,000\,\varphi_2 \,+\, 11\,340\,w_2 \,-\, 6{,}488\,1 \,=\, 0
\end{aligned}
$$

jednadžba rada na virtualnim pomacima:

(„vodeći" je virtualni pomak pomak $\delta\vec{w}_2$ koji odgovara neovisnom translacijskom pomaku $\vec{w}_2$, pa su plan (virtualnih) pomaka i dijagrami projekcija (virtualnih) pomaka na koordinatne osi jednaki planu i dijagramima prikazanima na stranicama 9 i 5, uz zamjene $w_2 \to \delta w_2$ i $\psi_{\text{i}} \to \delta\psi_{\text{i}}$, a veza je kutova $\delta\psi_{\{i,j\}}$ i orijentirane duljine δw_2 ista kao veza $\psi_{\{i,j\}}$ i w_2:

$$\delta\psi_{\{1,2\}} \,=\, -\frac{1}{5}\,\delta w_2 \,=\, -0{,}2\,\delta w_2 \qquad \& \qquad \delta\psi_{\{2,3\}} \,=\, \frac{\delta w_2}{3} \,=\, 0{,}333\,3\,\delta w_2\,\big)$$

$$
\begin{aligned}
&\big(M_{1,2} \,+\, M_{2,1}\big)\,\delta\psi_{\{1,2\}} \,+\, \big(M_{2,3} \,+\, M_{3,2}\big)\,\delta\psi_{\{2,3\}} \,=\, 0 \\
&\big(32\,400\,\varphi_2 \,+\, 19\,440\,w_2 \,+\, 64\,800\,\varphi_2 \,+\, 19\,440\,w_2\big) \cdot (-0{,}2\,\delta w_2) \\
&\qquad + \big(16\,200\,\varphi_2 \,-\, 8\,100\,w_2 \,-\, 6{,}488\,1 \\
&\qquad\quad + 8\,100\,\varphi_2 \,-\, 8\,100\,w_2 \,-\, 6{,}488\,1\big) \cdot 0{,}333\,3\,\delta w_2 \,=\, 0 \\
&\big(-11\,340\,\varphi_2 \,-\, 13\,176\,w_2 \,-\, 4{,}325\,4\big)\,\delta w_2 \,=\, 0 \quad \forall\,\delta w_2 \\
&-11\,340\,\varphi_2 \,-\, 13\,176\,w_2 \,-\, 4{,}325\,4 \,=\, 0 \quad \Big| \quad \times (-1) \\
&11\,340\,\varphi_2 \,+\, 13\,176\,w_2 \,+\, 4{,}325\,4 \,=\, 0
\end{aligned}
$$

sustav jednadžbi i njegovo rješenje:

$$
\begin{aligned}
&81\,000\,\varphi_2 \,+\, 11\,340\,w_2 \,-\, 6{,}488\,1 \,=\, 0 \\
&11\,340\,\varphi_2 \,+\, 13\,176\,w_2 \,+\, 4{,}325\,4 \,=\, 0 \qquad\qquad \text{(sustav je simetričan)}
\end{aligned}
$$

$$\varphi_2 \;=\; 0{,}000\,143\,3 \qquad \& \qquad w_2 \;=\; -0{,}000\,451\,6\ \mathrm{m}$$

vrijednosti momenata na krajevima štapova (treći (i posljednji) put):

$$M_{1,2} \;=\; 32\,400 \cdot 0{,}000\,143\,3 \;+\; 19\,440 \cdot (-0{,}000\,451\,6) \qquad\qquad =\; -4{,}136\ \mathrm{kNm}$$

$$M_{2,1} \;=\; 64\,800 \cdot 0{,}000\,143\,3 \;+\; 19\,440 \cdot (-0{,}000\,451\,6) \qquad\qquad =\; 0{,}507\ \mathrm{kNm}$$

$$M_{2,3} \;=\; 16\,200 \cdot 0{,}000\,143\,3 \;-\; 8\,100 \cdot (-0{,}000\,451\,6) \;-\; 6{,}488\,1 \;=\; -0{,}509\ \mathrm{kNm}$$

$$M_{3,2} \;=\; 8\,100 \cdot 0{,}000\,143\,3 \;-\; 8\,100 \cdot (-0{,}000\,451\,6) \;-\; 6{,}488\,1 \;=\; -1{,}669\ \mathrm{kNm}$$

(„Neravnoteža” momenata u čvoru 2 ($M_{2,1} \neq |M_{2,3}|$) i razlike u odnosu na vrijednosti momenata dobivene rješavanjem pomoću lijevoga osnovnog sistema posljedice su pogrešaka zaokruživanja u međukoracima proračuna i njihova „gomilanja”.)

dijagram momenata savijanja — na stranici 7 (uz lagahna prilagođavanja vrijednostī)