GS 2. — Prvi (i jedini) kolokvij (2024./2025.)

Zadatak 1. (Varijacija)

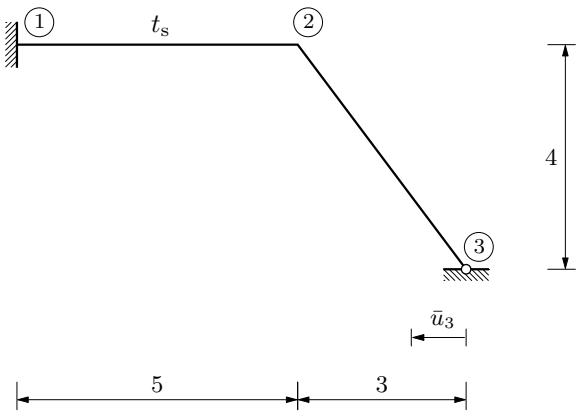
Inženjerskom metodom pomakā nacrtajte dijagram momenata savijanja!

$t_\mathrm{s} = -50^\circ\,\mathrm{C}$ (na štapu 1–2)

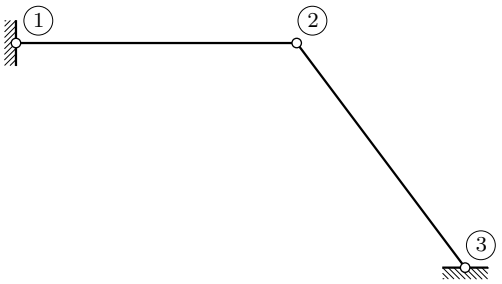
$\bar{u}_3 = 2{,}5\ \mathrm{mm}$

$EI = 20\,250\ \mathrm{kNm^2}$ sr

$\alpha_t = 1 \cdot 10^{-5}\ \mathrm{K^{-1}}$



zglobna shema i nepoznanice (zapravo, nepoznanica):

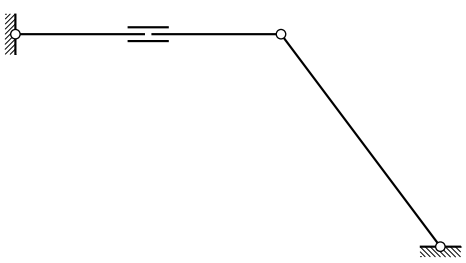


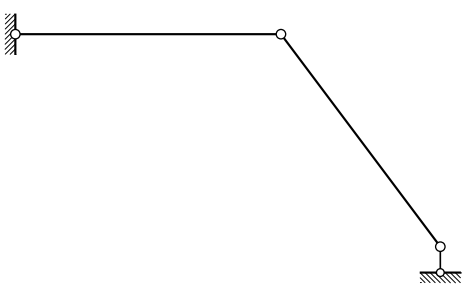
čvor 2 je s podlogom spojem s dva štapa osi kojih nisu na pravcu

$\Rightarrow$ čvor 2 je nepomičan

nepoznanica: (samo i jedino) φ_2

mehanizmi za utjecaj promjene temperature i za utjecaj prisilnoga pomaka:



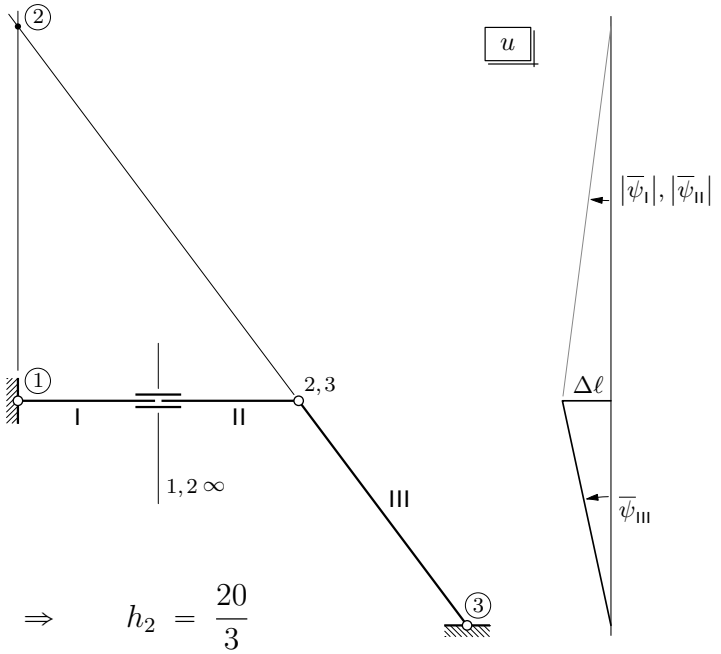


skraćenje štapa $\{1,2\}$ zbog jednolike promjene temperature:

$$\Delta\ell \;=\; \alpha_t\,|t_{\mathrm{s}}|\,\ell_{\{1,2\}} \;=\; 1\cdot 10^{-5}\cdot 50\cdot 5 \;=\; 0{,}002\,5$$

vrijednosti momenata upetosti za utjecaj promjene temperature:

izračunavanje kutova $\overline{\overline{\psi}}_{\{i,j\}}$ pomoću dijagrama projekcija pomakā na horizontalnu os:

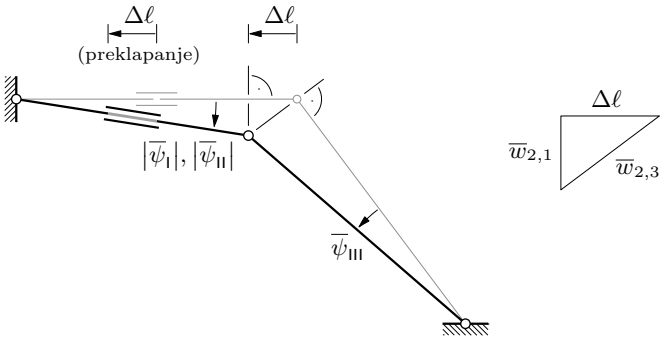


$$\frac{h_2}{5} \;=\; \frac{4}{3} \qquad\Rightarrow\qquad h_2 \;=\; \frac{20}{3}$$

$$\overline{\psi}_{\mathrm{I}} \;=\; -\frac{\Delta\ell}{h_2} \;=\; -0{,}000\,375 \;=\; \overline{\psi}_{\{1,2\}}$$

$$\overline{\psi}_{\mathrm{III}} \;=\; \frac{\Delta\ell}{4} \;=\; 0{,}000\,625 \;=\; \overline{\psi}_{\{2,3\}}$$

ili izračunavanje kutova $\overline{\psi}_{\{i,j\}}$ pomoću plana pomakā (zapetljanije, ali zornije):



$$\frac{\overline{w}_{2,1}}{\Delta\ell} \;=\; \frac{3}{4} \qquad\Rightarrow\qquad \overline{w}_{2,1} \;=\; \frac{3}{4}\,\Delta\ell \;=\; 0{,}001\,875$$

$$\frac{\overline{w}_{2,3}}{\Delta\ell} \;=\; \frac{5}{4} \qquad\Rightarrow\qquad \overline{w}_{2,3} \;=\; \frac{5}{4}\,\Delta\ell \;=\; 0{,}003\,125$$

$$\overline{\psi}_{\text{I}} \;=\; -\frac{\overline{w}_{2,1}}{\ell_{\{1,2\}}} \;=\; -0{,}000\,375 \;=\; \overline{\psi}_{\{1,2\}}$$

$$\overline{\psi}_{\text{III}} \;=\; \frac{\overline{w}_{2,3}}{\ell_{\{2,3\}}} \;=\; 0{,}000\,625 \;=\; \overline{\psi}_{\{2,3\}}$$

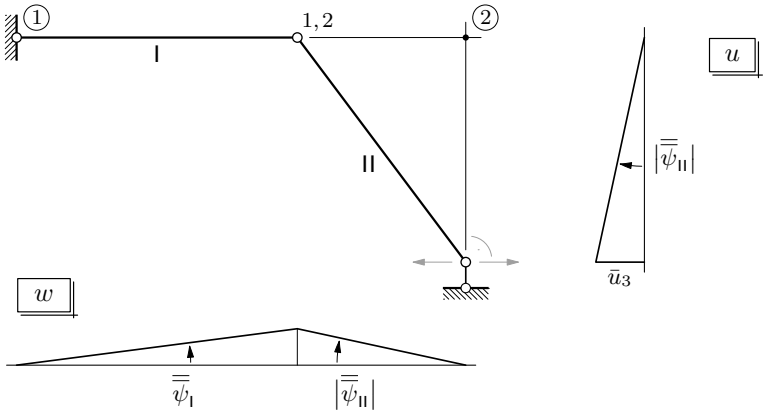
vrijednosti momenata upetosti:

$$\overline{M}_{1,2}\,(t_{\text{s}}) \;=\; \overline{M}_{2,1}\,(t_{\text{s}}) \;=\; -6\,k_{\{1,2\}}\,\overline{\psi}_{\{1,2\}} \;=\; -6\cdot\frac{20\,250}{5}\cdot(-0{,}000\,375) \;=\; 9{,}112\,5\ \text{kNm}$$

$$\overline{M}_{2,3}\,(t_{\text{s}}) \;=\; -3\,k_{\{2,3\}}\,\overline{\psi}_{\{2,3\}} \;=\; -3\cdot\frac{20\,250}{5}\cdot 0{,}000\,625 \;=\; -7{,}593\,75\ \text{kNm}$$

vrijednosti momenata upetosti za utjecaj prisilnoga pomaka:

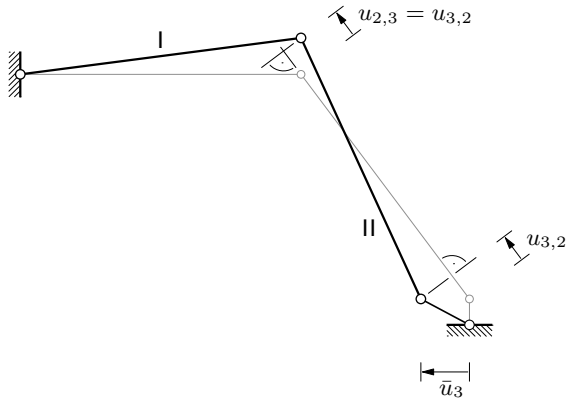
izračunavanje kutova $\overline{\psi}_{\{i,j\}}$ pomoću dijagramā projekcija pomakā na horizontalnu i vertikalnu os:



$$\overline{\overline{\psi}}_{\text{II}} \;=\; -\frac{\bar{u}_3}{4} \;=\; -0{,}000\,625 \;=\; \overline{\overline{\psi}}_{\{2,3\}}$$

$$\overline{\overline{\psi}}_{\text{I}}\cdot 5 \;=\; \left|\overline{\overline{\psi}}_{\text{II}}\right|\cdot 3 \qquad \Rightarrow \qquad \overline{\overline{\psi}}_{\text{I}} \;=\; \frac{3}{5}\left|\overline{\overline{\psi}}_{\text{II}}\right| \;=\; 0{,}000\,375 \;=\; \overline{\psi}_{\{1,2\}}$$

ili izračunavanje kutova $\overline{\overline{\psi}}_{\{i,j\}}$ pomoću plana pomakā (puno zapetljanije):



$\overline{\overline{\psi}}_{\text{I}}$ & $\overline{\overline{\psi}}_{\text{II}}$ — DZ! (za hrabr(ij)e)

vrijednosti momenata upetosti:

$$\overline{M}_{1,2}(\bar{u}_3) \;=\; \overline{M}_{2,1}(\bar{u}_3) \;=\; -6\,k_{\{1,2\}}\,\overline{\overline{\psi}}_{\{1,2\}} \;=\; -6\cdot\frac{20\,250}{5}\cdot 0{,}000\,375 \;=\; -9{,}112\,5\ \text{kNm}$$

$$\overline{M}_{2,3}(\bar{u}_3) \;=\; -3\,k_{\{2,3\}}\,\overline{\overline{\psi}}_{\{2,3\}} \;=\; -3\cdot\frac{20\,250}{5}\cdot(-0{,}000\,625) \;=\; 7{,}593\,75\ \text{kNm}$$

izrazi za vrijednosti ukupnih momenata na krajevima štapova:

$$M_{1,2} \;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;+\; \overline{M}_{1,2}(t_\text{s}) \;+\; \overline{M}_{1,2}(\bar{u}_3) \;=\; 2\,k_{\{1,2\}}\,\varphi_2 \;+\; 9{,}112\,5 \;-\; 9{,}112\,5 \;=\; 2\,k_{\{1,2\}}\,\varphi_2$$

$$M_{2,1} \;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;+\; \overline{M}_{2,1}(t_\text{s}) \;+\; \overline{M}_{2,1}(\bar{u}_3) \;=\; 4\,k_{\{1,2\}}\,\varphi_2 \;+\; 9{,}112\,5 \;-\; 9{,}112\,5 \;=\; 4\,k_{\{1,2\}}\,\varphi_2$$

$$M_{2,3} \;=\; 3\,k_{\{2,3\}}\,\varphi_2 \;+\; \overline{M}_{2,3}(t_\text{s}) \;+\; \overline{M}_{2,3}(\bar{u}_3) \;=\; 3\,k_{\{2,3\}}\,\varphi_2 \;-\; 7{,}593\,75 \;+\; 7{,}593\,75 \;=\; 3\,k_{\{2,3\}}\,\varphi_2$$

jednadžba ravnoteže momenata u čvoru 2:

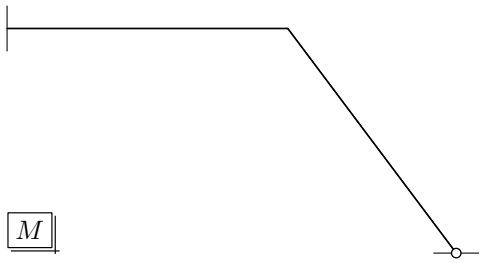
$$-M_{2,1} \;-\; M_{2,3} \;=\; 0 \qquad \text{odnosno} \qquad M_{2,1} \;+\; M_{2,3} \;=\; 0$$

$$4\,k_{\{1,2\}}\,\varphi_2 \;+\; 3\,k_{\{2,3\}}\,\varphi_2 \;=\; 0$$

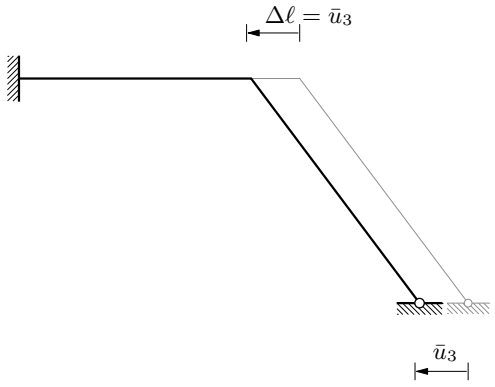
$$\left(4\,k_{\{1,2\}} \;+\; 3\,k_{\{2,3\}}\right)\varphi_2 \;=\; 0 \qquad \Rightarrow \qquad \varphi_2 \;=\; 0$$

$$\Rightarrow \qquad M_{1,2} \;=\; M_{2,1} \;=\; M_{2,3} \;=\; 0$$

dijagram momenata savijanja:



skica „progibne linije” (zora radi):



[DZ — riješite zadatak ako je $t_\text{s} \;=\; +50^\circ\,\text{C}$!]