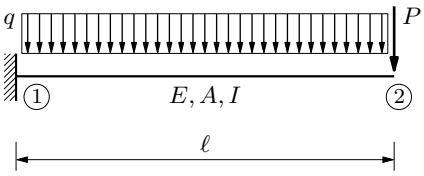
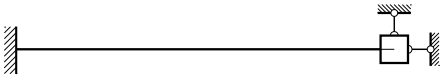
Opća metoda pomaka — konzola





$u_1 = w_1 = \varphi_1 = 0$

nepoznanice: u_2, w_2, φ_2

veza duljina pomakā i kutova zaokreta krajeva štapa i duljina pomakā i kutova zaokreta čvorova:

$$u_{1,2} = u_1 \qquad u_{2,1} = u_2$$
$$w_{1,2} = w_1 \qquad w_{2,1} = w_2$$
$$\varphi_{1,2} = \varphi_1 \qquad \varphi_{2,1} = \varphi_2$$

izrazi za vrijednosti sila upetosti (odjeljak *12.3.2. Sile upetosti* na stranici 282 skripata [http://master.grad.hr/nastava/gs/gs1/gs1.pdf] uz $b_\ell = b_\mathrm{d} = \ell/2$):

$$\overline{N}_{1,2} = 0 \qquad \overline{N}_{2,1} = 0$$
$$\overline{T}_{1,2} = -\frac{q\,\ell}{2} \qquad \overline{T}_{2,1} = -\frac{q\,\ell}{2}$$
$$\overline{M}_{1,2} = \frac{q\,\ell^2}{12} \qquad \overline{M}_{2,1} = -\frac{q\,\ell^2}{12}$$

izrazi za vrijednosti sila u stanju prisilnih pomaka (odgovarajuće komponente matrice krutosti ravnoga štapa navedene u odjeljku *12.3.3. Odnos između sila na krajevima i pomakā krajeva* na stranici 285 skripata, uz $k^f = k$):

$$n_{1,2} = -k^a\,u_2 \qquad n_{2,1} = k^a\,u_2$$
$$t_{1,2} = -\frac{12\,k}{\ell^2}\,w_2 - \frac{6\,k}{\ell}\,\varphi_2 \qquad t_{1,2} = \frac{12\,k}{\ell^2}\,w_2 + \frac{6\,k}{\ell}\,\varphi_2$$
$$m_{1,2} = \frac{6\,k}{\ell}\,w_2 + 2\,k\,\varphi_2 \qquad m_{1,2} = \frac{6\,k}{\ell}\,w_2 + 4\,k\,\varphi_2$$

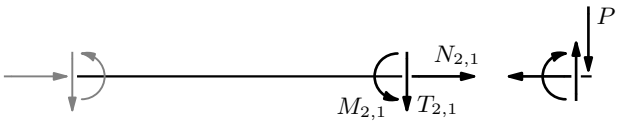
izrazi za (ukupne) vrijednosti sila na krajevima:

$$N_{1,2} = \overline{N}_{1,2} + n_{1,2} = -k^a\,u_2, \qquad N_{2,1} = \overline{N}_{2,1} + n_{2,1} = k^a\,u_2$$
$$T_{1,2} = \overline{T}_{1,2} + t_{1,2} = -\frac{q\,\ell}{2} - \frac{12\,k}{\ell^2}\,w_2 - \frac{6\,k}{\ell}\,\varphi_2$$
$$T_{2,1} = \overline{T}_{1,2} + t_{1,2} = -\frac{q\,\ell}{2} + \frac{12\,k}{\ell^2}\,w_2 + \frac{6\,k}{\ell}\,\varphi_2$$

$$M_{1,2} \;=\; \overline{M}_{1,2} \,+\, m_{1,2} \;=\; \frac{q\,\ell^2}{12} \,+\, \frac{6\,k}{\ell}\,w_2 \,+\, 2\,k\,\varphi_2$$

$$M_{2,1} \;=\; \overline{M}_{1,2} \,+\, m_{1,2} \;=\; -\frac{q\,\ell^2}{12} \,+\, \frac{6\,k}{\ell}\,w_2 \,+\, 4\,k\,\varphi_2$$

jednadžbe ravnoteže sile u čvoru 2:



$$-N_{2,1} \;=\; 0, \qquad N_{2,1} \;=\; 0: \qquad k^a\,u_2 \;=\; 0 \qquad \Rightarrow \qquad u_2 \;=\; 0 \qquad \Rightarrow \qquad N_{1,2} \;=\; N_{2,1} \;=\; 0$$

$$-T_{2,1} \,+\, P \;=\; 0, \qquad T_{2,1} \,-\, P \;=\; 0: \qquad -\frac{q\,\ell}{2} \,+\, \frac{12\,k}{\ell^2}\,w_2 \,+\, \frac{6\,k}{\ell}\,\varphi_2 \,-\, P \;=\; 0$$

$$-M_{2,1} \;=\; 0, \qquad M_{2,1} \;=\; 0: \qquad -\frac{q\,\ell^2}{12} \,+\, \frac{6\,k}{\ell}\,w_2 \,+\, 4\,k\,\varphi_2 \;=\; 0$$

sustav jednadžbi:

$$\frac{12\,k}{\ell^2}\,w_2 \,+\, \frac{6\,k}{\ell}\,\varphi_2 \;=\; \frac{q\,\ell}{2} \,+\, P \qquad (\text{✍})$$

$$\frac{6\,k}{\ell}\,w_2 \,+\, 4\,k\,\varphi_2 \;=\; \frac{q\,\ell^2}{12} \qquad (\text{✍})$$

rješavanje sustava (i rješenje):

$$\varphi_2 \;=\; \frac{q\,\ell^2}{48\,k} \,-\, \frac{3}{2\,\ell}\,w_2 \qquad (\text{iz } (\text{✍}))$$

$$\frac{12\,k}{\ell^2}\,w_2 \,+\, \frac{6\,k}{\ell}\left(-\frac{3}{2\,\ell}\,w_2 \,+\, \frac{q\,\ell^2}{48\,k}\right) \;=\; \frac{q\,\ell}{2} \,+\, P \qquad (\text{u } (\text{✍}))$$

$$\left(\frac{12\,k}{\ell^2} \,-\, \frac{9\,k}{\ell^2}\right) w_2 \;=\; \frac{q\,\ell}{2} \,+\, P \,-\, \frac{q\,\ell}{8}$$

$$\frac{3\,k}{\ell^2}\,w_2 \;=\; \frac{3\,q\,\ell}{8} \,+\, P \qquad \Rightarrow \qquad w_2 \;=\; \frac{\ell^2}{3\,k}\left(\frac{3\,q\,\ell}{8} \,+\, P\right)$$

$$\varphi_2 \;=\; \frac{q\,\ell^2}{48\,k} \,-\, \frac{3}{2\,\ell}\,\frac{\ell^2}{3\,k}\left(\frac{3\,q\,\ell}{8} \,+\, P\right) \;=\; \frac{q\,\ell^2}{48\,k} \,-\, \frac{3\,q\,\ell^2}{16\,k} \,-\, \frac{P\ell}{2\,k} \;=\; -\frac{q\,\ell^2}{6\,k} \,-\, \frac{P\ell}{2\,k}$$

vrijednosti sila na krajevima:

$$T_{1,2} \;=\; -\frac{q\,\ell}{2} \,-\, \frac{12\,k}{\ell^2}\,\frac{\ell^2}{3\,k}\left(\frac{3\,q\,\ell}{8} \,+\, P\right) \,-\, \frac{6\,k}{\ell}\left(-\frac{q\,\ell^2}{6\,k} \,-\, \frac{P\ell}{2\,k}\right)$$

$$\;=\; -\frac{q\,\ell}{2} \,-\, \frac{3\,q\,\ell}{2} \,-\, 4\,P \,+\, q\,\ell \,+\, 3\,P \;=\; -q\,\ell \,-\, P$$

$$
\begin{aligned}
T_{2,1} &= -\frac{q\,\ell}{2} + \frac{12\,k}{\ell^2}\,\frac{\ell^2}{3\,k}\left(\frac{3\,q\,\ell}{8} + P\right) + \frac{6\,k}{\ell}\left(-\frac{q\,\ell^2}{6\,k} - \frac{P\ell}{2\,k}\right) \\
&= -\frac{q\,\ell}{2} + \frac{3\,q\,\ell}{2} + 4\,P - q\,\ell - 3\,P = P
\end{aligned}
$$

$$
\begin{aligned}
M_{1,2} &= \frac{q\,\ell^2}{12} + \frac{6\,k}{\ell}\,\frac{\ell^2}{3\,k}\left(\frac{3\,q\,\ell}{8} + P\right) + 2\,k\left(-\frac{q\,\ell^2}{6\,k} - \frac{P\ell}{2\,k}\right) \\
&= \frac{q\,\ell^2}{12} + \frac{3\,q\,\ell^2}{4} + 2\,P\ell - \frac{1\,q\,\ell^2}{3} - P\ell = \frac{q\,\ell^2}{2} + P\ell
\end{aligned}
$$

$$
\begin{aligned}
M_{2,1} &= -\frac{q\,\ell^2}{12} + \frac{6\,k}{\ell}\,\frac{\ell^2}{3\,k}\left(\frac{3\,q\,\ell}{8} + P\right) + 4\,k\left(-\frac{q\,\ell^2}{6\,k} - \frac{P\ell}{2\,k}\right) \\
&= -\frac{q\,\ell^2}{12} + \frac{3\,q\,\ell^2}{4} + 2\,P\ell - \frac{2\,q\,\ell^2}{3} - 2\,P\ell = 0
\end{aligned}
$$