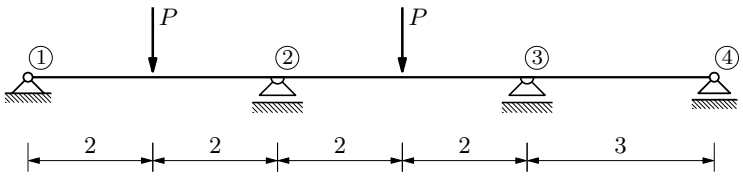
Opća metoda pomaka — kontinuirani nosač



$P = 100\,\mathrm{kN}$

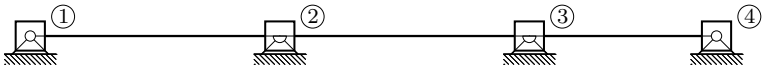
$EI = \text{const.}$

$w_1 = w_2 = w_3 = w_4 = 0 \quad \Longleftarrow$ ležajevi (trivijalna kinematička kondenzacija)

$u_1 = u_2 = u_3 = u_4 = 0 \quad \Longleftarrow$ $u_1 = 0 \Leftarrow$ ležaj, ostalo: $N = 0 \Rightarrow \varepsilon = 0$ (kinematička kondenzacija)

nepoznanice: $\varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4$

osnovni sistem za opću metodu pomaka:



$$\varphi_{1,2} = \varphi_1, \qquad \varphi_{2,1} = \varphi_2$$
$$\varphi_{2,3} = \varphi_2, \qquad \varphi_{3,2} = \varphi_3$$
$$\varphi_{3,4} = \varphi_3, \qquad \varphi_{4,3} = \varphi_4$$

izrazi za vrijednosti momenata na krajevima štapova:

$$M_{i,j} = \overline{M}_{j,i} + m_{i,j}$$

$$\overline{M}_{i,j} = -\overline{M}_{j,i} = \frac{P\,\ell_{\{i,j\}}}{8}$$

$$m_{i,j} = m_{i,j}(\varphi_i) + m_{i,j}(\varphi_j) = 4\,k_{\{i,j\}}\,\varphi_i + 2\,k_{\{i,j\}}\,\varphi_j$$

$$M_{1,2} = \frac{P\cdot 4}{8} + 4\,k_{\{1,2\}}\,\varphi_1 + 2\,k_{\{1,2\}}\,\varphi_2 = 50 + EI\,\varphi_1 + \frac{1}{2}EI\,\varphi_2$$

$$M_{2,1} = -\frac{P\cdot 4}{8} + 2\,k_{\{1,2\}}\,\varphi_1 + 4\,k_{\{1,2\}}\,\varphi_2 = -50 + \frac{1}{2}EI\,\varphi_1 + EI\,\varphi_2$$

$$M_{2,3} = \frac{P\cdot 4}{8} + 4\,k_{\{2,3\}}\,\varphi_2 + 2\,k_{\{2,3\}}\,\varphi_3 = 50 + EI\,\varphi_2 + \frac{1}{2}EI\,\varphi_3$$

$$M_{3,2} = -\frac{P\cdot 4}{8} + 2\,k_{\{2,3\}}\,\varphi_2 + 4\,k_{\{2,3\}}\,\varphi_3 = -50 + \frac{1}{2}EI\,\varphi_2 + EI\,\varphi_3$$

$$M_{3,4} = 4\,k_{\{3,4\}}\,\varphi_3 + 2\,k_{\{3,4\}}\,\varphi_4 = \frac{4}{3}EI\,\varphi_3 + \frac{2}{3}EI\,\varphi_4$$

$$M_{4,3} = 2\,k_{\{3,4\}}\,\varphi_3 + 4\,k_{\{3,4\}}\,\varphi_4 = \frac{2}{3}EI\,\varphi_3 + \frac{4}{3}EI\,\varphi_4$$

jednadžbe ravnoteže momenata u čvorovima:

$$-M_{1,2} = 0 \qquad \rightarrow \qquad M_{1,2} = 0$$

$$50 + EI\,\varphi_1 + \frac{1}{2}EI\,\varphi_2 = 0$$

$$\boxed{\varphi_1 + \frac{1}{2}\varphi_2 = -\frac{50}{EI}}$$

$$-M_{2,1} - M_{2,3} = 0 \qquad \rightarrow \qquad M_{2,1} + M_{2,3} = 0$$

$$-50 + \frac{1}{2}EI\,\varphi_1 + EI\,\varphi_2 + 50 + EI\,\varphi_2 + \frac{1}{2}EI\varphi_3 = 0$$

$$\boxed{\frac{1}{2}\varphi_1 + 2\,\varphi_2 + \frac{1}{2}\varphi_3 = 0}$$

$$-M_{3,2} - M_{3.4} = 0 \qquad \rightarrow \qquad M_{3,2} + M_{3,4} = 0$$

$$-50 + \frac{1}{2}EI\,\varphi_2 + EI\,\varphi_3 + \frac{4}{3}EI\,\varphi_3 + \frac{2}{3}EI\,\varphi_4 = 0$$

$$\boxed{\frac{1}{2}\varphi_2 + \frac{7}{3}\varphi_3 + \frac{2}{3}\varphi_4 = \frac{50}{EI}}$$

$$-M_{4,3} = 0 \qquad \rightarrow \qquad M_{4,3} = 0$$

$$\boxed{\frac{2}{3}\varphi_3 + \frac{4}{3}\varphi_4 = 0}$$

rješenje sustava:

$$\varphi_1 = -\frac{700}{13\,EI}, \qquad \varphi_2 = \frac{100}{13\,EI}, \qquad \varphi_3 = \frac{300}{13\,EI}, \qquad \varphi_4 = -\frac{150}{13\,EI}$$

vrijednosti momenata na krajevima:

$$M_{1,2} = 50 + EI\left(-\frac{700}{13\,EI}\right) + \frac{1}{2}EI\left(\frac{100}{13\,EI}\right) = 0$$

$$M_{2,1} = -50 + \frac{1}{2}EI\left(-\frac{700}{13\,EI}\right) + EI\left(\frac{100}{13\,EI}\right) = -\frac{900}{13} = -69{,}23$$

$$M_{2,3} = 50 + EI\left(\frac{100}{13\,EI}\right) + \frac{1}{2}EI\left(\frac{300}{13\,EI}\right) = \frac{900}{13} = 69{,}23$$

$$M_{3,2} = -50 + \frac{1}{2}EI\left(\frac{100}{13\,EI}\right) + EI\left(\frac{300}{13\,EI}\right) = -\frac{300}{13} = -23{,}08$$

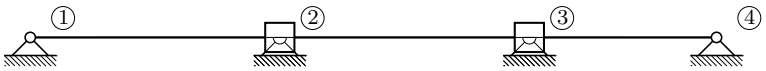
$$M_{3,4} \;=\; \frac{4}{3}\,EI\left(\frac{300}{13\,EI}\right) \;+\; \frac{2}{3}\,EI\left(-\frac{150}{13\,EI}\right) \;=\; \frac{300}{13} \;=\; 23{,}08$$

$$M_{4,3} \;=\; \frac{2}{3}\,EI\left(\frac{300}{13\,EI}\right) \;+\; \frac{4}{3}\,EI\left(-\frac{150}{13\,EI}\right) \;=\; 0$$

još jednom, primjenom statičke kondenzacije:

nepoznanice: $\varphi_2,\ \varphi_3$

osnovni sistem za opću metodu pomaka:



izrazi za vrijednosti momenata na krajevima štapova:

$$M^{\mathrm{c}}_{2,1} \;=\; \overline{M}^{\mathrm{c}}_{2,1} \;+\; m^{\mathrm{c}}_{2,1}(\varphi_2) \;=\; \left(\overline{M}_{2,1} \;-\; \frac{1}{2}\,\overline{M}_{1,2}\right) + \left(m_{2,1}(\varphi_2) \;-\; \frac{1}{2}\,m_{1,2}(\varphi_2)\right)$$

$$= \left(-\frac{P\,\ell_{\{1,2\}}}{8} \;-\; \frac{1}{2}\,\frac{P\,\ell_{\{1,2\}}}{8}\right) + \left(4\,k_{\{1,2\}}\,\varphi_2 \;-\; \frac{1}{2}\cdot 2\,k_{\{1,2\}}\,\varphi_2\right)$$

$$= \;-\frac{3}{16}\,P\,\ell_{\{1,2\}} \;+\; 3\,k_{\{1,2\}}\,\varphi_2 \;=\; -75 \;+\; \frac{3}{4}\,EI\,\varphi_2$$

$$M_{2,3} \;=\; 50 \;+\; EI\,\varphi_2 \;+\; \frac{1}{2}\,EI\,\varphi_3$$

$$M_{3,2} \;=\; -50 \;+\; \frac{1}{2}\,EI\,\varphi_2 \;+\; EI\,\varphi_3$$

$$M^{\mathrm{c}}_{3,4} \;=\; m^{\mathrm{c}}_{3,4}(\varphi_3) \;=\; m_{3,4}(\varphi_3) \;-\; \frac{1}{2}\,m_{4,3}(\varphi_3) \;=\; 3\,k_{\{3,4\}}\,\varphi_3 \;=\; EI\,\varphi_3$$

jednadžbe ravnoteže momenata u čvorovima:

$$-M_{2,1} \;-\; M_{2,3} \;=\; 0 \quad\rightarrow\quad M_{2,1} \;+\; M_{2,3} \;=\; 0 \quad\rightarrow\quad M^{\mathrm{c}}_{2,1} \;+\; M_{2,3} \;=\; 0$$

$$-\,75 \;+\; \frac{3}{4}\,EI\,\varphi_2 \;+\; 50 \;+\; EI\,\varphi_2 \;+\; \frac{1}{2}\,EI\varphi_3 \;=\; 0$$

$$\boxed{\frac{7}{4}\,\varphi_2 \;+\; \frac{1}{2}\,\varphi_3 \;=\; \frac{25}{EI}}$$

$$-M_{3,2} \;-\; M_{3.4} \;=\; 0 \quad\rightarrow\quad M_{3,2} \;+\; M_{3,4} \;=\; 0 \quad\rightarrow\quad M_{3,2} \;+\; M^{\mathrm{c}}_{3,4} \;=\; 0$$

$$-50 \;+\; \frac{1}{2}\,EI\,\varphi_2 \;+\; EI\,\varphi_3 \;+\; EI\,\varphi_3 \;=\; 0$$

$$\boxed{\frac{1}{2}\,\varphi_2 \;+\; 2\,\varphi_3 \;=\; \frac{50}{EI}}$$

rješenje sustava:

$$\varphi_2 \;=\; \frac{100}{13\,EI}, \qquad \varphi_3 \;=\; \frac{300}{13\,EI}$$

vrijednosti momenata na krajevima štapova:

$$M_{1,2} \;=\; 0$$

$$M^{\mathrm{c}}_{2,1} \;=\; -75 \;+\; \frac{3}{4}\,EI\left(\frac{100}{13\,EI}\right) \;=\; -\frac{900}{13} \;=\; -69{,}23$$

$$M_{2,3} \;=\; 69{,}23$$

$$M_{3,2} \;=\; -23{,}08$$

$$M^{\mathrm{c}}_{3,4} \;=\; EI\left(\frac{300}{13\,EI}\right) \;=\; 23{,}08$$

$$M_{4,3} \;=\; 0$$

[Domaća zabava: nacrtajte dijagram momenata savijanja!]