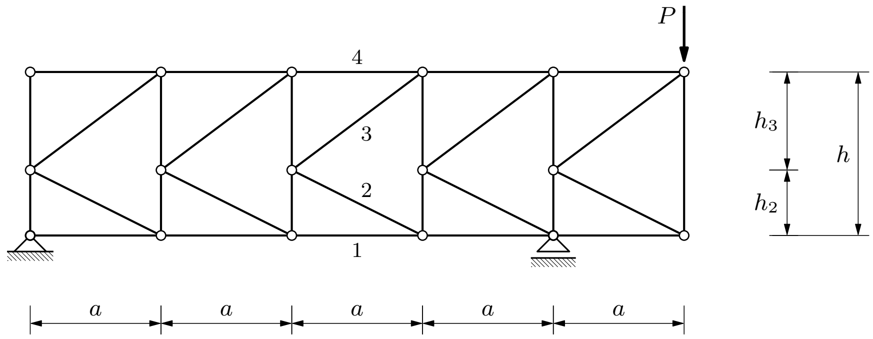
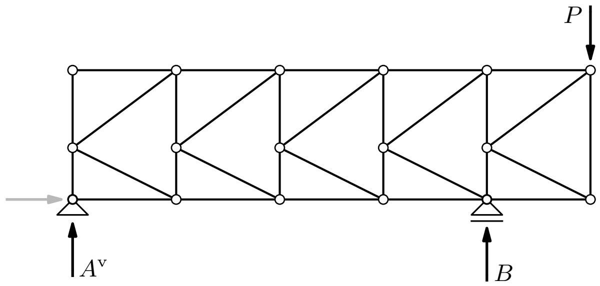
K–rešetka – Ritterov postupak

Ritterovim postupkom odredite vrijednosti sila u štapovima 1, 2, 3 i 4!



vrijednosti reakcija:

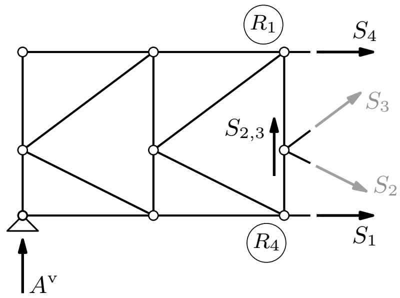


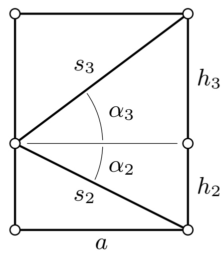
$$\sum\nolimits_{\text{cijeli}} M_{/B} = 0 : \qquad -4\,a\,A^{v} - a\,P = 0 \qquad \Rightarrow \qquad A^{v} = -\frac{1}{4}\,P$$

$$\sum\nolimits_{\text{cijeli}} M_{/A} = 0 : \qquad 4\,a\,B - 5\,a\,P = 0 \qquad \Rightarrow \qquad B = \frac{5}{4}\,P$$

provjera: $\sum\nolimits_{\text{cijeli}} F_z = -A^{v} - B + P = -\left(-\frac{1}{4}\,P\right) - \frac{5}{4}\,P + P = 0$

vrijednosti sila u štapovima:





$$\sum\nolimits_{\text{lijevi dio}} M_{/R_1} = 0 : \qquad -2\,a\,A^{v} + h\,S_1 = 0$$

$$\Rightarrow \qquad S_1 = \frac{2\,a}{h}\,A^{v} = \frac{2\,a}{h}\left(-\frac{1}{4}\,P\right) = -\frac{a}{2\,h}\,P$$

$$\sum\nolimits_{\text{lijevi dio}} M_{/R_4} \; = \; 0 \; : \qquad -2\,a\,A^{\mathrm{v}} \; - \; h\,S_4 \; = \; 0$$

$$\Rightarrow \qquad S_4 \; = \; -\frac{2\,a}{h}\,A^{\mathrm{v}} \; = \; -\frac{2\,a}{h}\left(-\frac{1}{4}\,P\right) \; = \; \frac{a}{2\,h}\,P$$

$$\sum\nolimits_{\text{lijevi dio}} F_z \; = \; 0 \; : \qquad -A^{\mathrm{v}} \; - \; S_{2,3} \; = \; 0 \qquad \Rightarrow \qquad S_{2,3} \; = \; -A^{\mathrm{v}} \; = \; -\left(-\frac{1}{4}\,P\right) \; = \; \frac{1}{4}\,P$$

provjera: očito je $\sum\nolimits_{\text{lijevi dio}} F_x \; = \; S_1 \; + \; S_4 \; = \; 0$

$$\sum\nolimits_{\text{lijevi dio}} M_{/A} \; = \; -h\,S_4 \; + \; 2\,a\,S_{2,3} \; = \; -h\left(\frac{a}{2\,h}\,P\right) + \; 2\,a\left(\frac{1}{4}\,P\right) \; = \; 0$$

$$\left.\begin{aligned} S_2^{\mathrm{h}} \; + \; S_3^{\mathrm{h}} \; &= \; 0 \\ S_2^{\mathrm{v}} \; - \; S_3^{\mathrm{v}} \; &= \; -S_{2,3} \end{aligned}\right\}$$

$$\frac{S_2^{\mathrm{v}}}{S_2^{\mathrm{h}}} \; = \; \frac{h_2}{a} \qquad \Rightarrow \qquad S_2^{\mathrm{v}} \; = \; \frac{h_2}{a}\,S_2^{\mathrm{h}} \qquad \& \qquad \frac{S_3^{\mathrm{v}}}{S_3^{\mathrm{h}}} \; = \; \frac{h_3}{a} \qquad \Rightarrow \qquad S_3^{\mathrm{v}} \; = \; \frac{h_3}{a}\,S_3^{\mathrm{h}}$$

$$\left.\begin{aligned} S_2^{\mathrm{h}} \; + \; \quad S_3^{\mathrm{h}} \; &= \; 0 \\ \frac{h_2}{a}\,S_2^{\mathrm{h}} \; - \; \frac{h_3}{a}\,S_3^{\mathrm{h}} \; &= \; -S_{2,3} \end{aligned}\right\}$$

$$S_3^{\mathrm{h}} \; = \; -S_2^{\mathrm{h}}$$

$$\frac{h_2}{a}\,S_2^{\mathrm{h}} \; - \; \frac{h_3}{a}\left(-S_2^{\mathrm{h}}\right) \; = \; -S_{2,3}$$

$$\left(\frac{h_2}{a} \; + \; \frac{h_3}{a}\right) S_2^{\mathrm{h}} \; = \; -S_{2,3}$$

$$S_2^{\mathrm{h}} \; = \; -\frac{a}{h_2 + h_3}\,S_{2,3} \; = \; -\frac{a}{h}\,S_{2,3} \qquad \& \qquad S_3^{\mathrm{h}} \; = \; \frac{a}{h}\,S_{2,3}$$

$$S_2^{\mathrm{v}} \; = \; -\frac{h_2}{a}\,\frac{a}{h}\,S_{2,3} \; = \; -\frac{h_2}{h}\,S_{2,3} \qquad \& \qquad S_3^{\mathrm{v}} \; = \; \frac{h_3}{h}\,S_{2,3}$$

provjera: $S_2^{\mathrm{v}} \; - \; S_3^{\mathrm{v}} \; = \; -\frac{h_2}{h}\,S_{2,3} \; - \; \frac{h_3}{h}\,S_{2,3} \; = \; -\frac{h_2 + h_3}{h}\,S_{2,3} \; = \; -S_{2,3}$

$$S_2 \; = \; -\sqrt{\left(S_2^{\mathrm{h}}\right)^2 \; + \; \left(S_2^{\mathrm{v}}\right)^2} \; = \; -\sqrt{\frac{a^2 + h_2^2}{h^2}\,S_{2,3}^2} \; = \; -\frac{\sqrt{a^2 + h_2^2}}{h}\,S_{2,3} \; = \; -\frac{\sqrt{a^2 + h_2^2}}{4\,h}\,P$$

$$S_3 \; = \; \sqrt{\left(S_3^{\mathrm{h}}\right)^2 \; + \; \left(S_3^{\mathrm{v}}\right)^2} \; = \; \frac{\sqrt{a^2 + h_3^2}}{4\,h}\,P$$

kako su ${S_2^{\mathrm{v}}}/{S_2^{\mathrm{h}}} = \operatorname{tg}\alpha_2$ i ${S_3^{\mathrm{v}}}/{S_3^{\mathrm{h}}} = \operatorname{tg}\alpha_3$, prethodni se izvod izrazā za S_2 i S_3 može „prevesti" u

$$\left.\begin{aligned} S_2^{\mathrm{h}} + S_3^{\mathrm{h}} &= 0 \\ S_2^{\mathrm{v}} - S_3^{\mathrm{v}} &= -S_{2,3} \end{aligned}\right\}$$

$$S_2^{\mathrm{v}} = S_2^{\mathrm{h}} \operatorname{tg}\alpha_2 \quad \& \quad S_3^{\mathrm{v}} = S_3^{\mathrm{h}} \operatorname{tg}\alpha_3$$

$$\left.\begin{aligned} S_2^{\mathrm{h}} + S_3^{\mathrm{h}} &= 0 \\ S_2^{\mathrm{h}} \operatorname{tg}\alpha_2 - S_3^{\mathrm{h}} \operatorname{tg}\alpha_3 &= -S_{2,3} \end{aligned}\right\}$$

$$S_3^{\mathrm{h}} = -S_2^{\mathrm{h}}$$

$$S_2^{\mathrm{h}} \operatorname{tg}\alpha_2 - (-S_2^{\mathrm{h}}) \operatorname{tg}\alpha_3 = -S_{2,3}$$

$$S_2^{\mathrm{h}} (\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3) = -S_{2,3}$$

$$S_2^{\mathrm{h}} = -\frac{1}{\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3} S_{2,3} \quad \& \quad S_3^{\mathrm{h}} = \frac{1}{\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3} S_{2,3}$$

$$S_2^{\mathrm{v}} = -\frac{\operatorname{tg}\alpha_2}{\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3} S_{2,3} \quad \& \quad S_3^{\mathrm{v}} = \frac{\operatorname{tg}\alpha_3}{\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3} S_{2,3}$$

$$S_2 = -\sqrt{\left(S_2^{\mathrm{h}}\right)^2 + \left(S_2^{\mathrm{v}}\right)^2} = -\sqrt{\frac{1 + \operatorname{tg}^2\alpha_2}{(\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3)^2} S_{2,3}^2} = -\frac{\sqrt{1 + \operatorname{tg}^2\alpha_2}}{\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3} S_{2,3}$$

$$S_3 = \sqrt{\left(S_3^{\mathrm{h}}\right)^2 + \left(S_3^{\mathrm{v}}\right)^2} = \frac{\sqrt{1 + \operatorname{tg}^2\alpha_3}}{\operatorname{tg}\alpha_2 + \operatorname{tg}\alpha_3} S_{2,3}$$

$$\left[\ \operatorname{tg}\alpha_2 = \frac{h_2}{a} \quad \& \quad \operatorname{tg}\alpha_3 = \frac{h_3}{a} \quad \Rightarrow \quad S_2 = \cdots \quad \& \quad S_3 = \cdots\ \right]$$

još jedan način izvođenja izraza za S_2 i S_3:

$$\left.\begin{aligned} S_2^{\mathrm{h}} + S_3^{\mathrm{h}} &= 0 \\ S_2^{\mathrm{v}} - S_3^{\mathrm{v}} &= -S_{2,3} \end{aligned}\right\}$$

$$\frac{S_2^{\mathrm{h}}}{S_2} = \frac{a}{s_2} \quad \Rightarrow \quad S_2^{\mathrm{h}} = \frac{a}{s_2} S_2 \quad \& \quad \frac{S_3^{\mathrm{h}}}{S_3} = \frac{a}{s_3} \quad \Rightarrow \quad S_3^{\mathrm{h}} = \frac{a}{s_3} S_3$$

$$\frac{S_2^{\mathrm{v}}}{S_2} = \frac{h_2}{s_2} \quad \Rightarrow \quad S_2^{\mathrm{v}} = \frac{h_2}{s_2} S_2 \quad \& \quad \frac{S_3^{\mathrm{v}}}{S_3} = \frac{h_3}{s_3} \quad \Rightarrow \quad S_3^{\mathrm{v}} = \frac{h_3}{s_3} S_3$$

$$\left.\begin{aligned} \frac{a}{s_2} S_2 + \frac{a}{s_3} S_3 &= 0 \\ \frac{h_2}{s_2} S_2 - \frac{h_3}{s_3} S_3 &= -S_{2,3} \end{aligned}\right\}$$

$$\frac{a}{s_3}\,S_3 \;=\; -\frac{a}{s_2}\,S_2 \qquad\Rightarrow\qquad S_3 \;=\; -\frac{s_3}{s_2}\,S_2$$

$$\frac{h_2}{s_2}\,S_2 \;-\; \frac{h_3}{s_3}\left(-\frac{s_3}{s_2}\,S_2\right) \;=\; -S_{2,3}$$

$$\left(\frac{h_2}{s_2} \;+\; \frac{h_3}{s_2}\right) S_2 \;=\; -S_{2,3}$$

$$S_2 \;=\; -\frac{s_2}{h_2+h_3}\,S_{2,3} \;=\; -\frac{s_2}{h}\,S_{2,3}$$

$$S_3 \;=\; -\frac{s_3}{s_2}\left(-\frac{s_2}{h}\,S_{2,3}\right) \;=\; \frac{s_3}{h}\,S_{2,3}$$

$$s_2 \;=\; \sqrt{a^2+h_2^2} \qquad \& \qquad s_3 \;=\; \sqrt{a^2+h_3^2} \qquad \left(\& \qquad S_{2,3} \;=\; \frac{1}{4}\,P\right)$$

$$S_2 \;=\; -\frac{\sqrt{a^2+h_2^2}}{4h}\,P \qquad \& \qquad S_3 \;=\; \frac{\sqrt{a^2+h_3^2}}{4h}\,P$$

provjera: uvrštavanjem u $\frac{h_2}{s_2}\,S_2 - \frac{h_3}{s_3}\,S_3 = -S_{2,3}$ može se pokazati da S_2 i S_3 zadovoljavaju tu jednadžbu [domaća zabava!]

kako su $S_i^{\mathrm{h}}/S_i = \cos\alpha_i$ i $S_i^{\mathrm{v}}/S_i = \sin\alpha_i$, netom provedeni izvod izrazā za S_2 i S_3 može se „prevesti” u

$$\left.\begin{aligned} S_2^{\mathrm{h}} \;+\; S_3^{\mathrm{h}} \;&=\; 0 \\ S_2^{\mathrm{v}} \;-\; S_3^{\mathrm{v}} \;&=\; -S_{2,3} \end{aligned}\right\}$$

$$S_i^{\mathrm{h}} \;=\; S_i\,\cos\alpha_i \qquad \& \qquad S_i^{\mathrm{v}} \;=\; S_i\,\sin\alpha_3$$

$$\left.\begin{aligned} S_2\,\cos\alpha_2 \;+\; S_3\,\cos\alpha_3 \;&=\; 0 \\ S_2\,\sin\alpha_2 \;-\; S_3\,\sin\alpha_3 \;&=\; -S_{2,3} \end{aligned}\right\}$$

$$S_3 \;=\; -\frac{\cos\alpha_2}{\cos\alpha_3}\,S_2$$

$$S_2\left[\sin\alpha_2 \;-\; \left(-\frac{\cos\alpha_2}{\cos\alpha_3}\right)\sin\alpha_3\right] \;=\; -S_{2,3}$$

$$S_2 \;=\; -\frac{S_{2,3}}{\sin\alpha_2 \;+\; \cos\alpha_2\;\mathrm{tg}\,\alpha_3}$$

$$S_3 \;=\; \frac{S_{2,3}}{\sin\alpha_3 \;+\; \cos\alpha_3\;\mathrm{tg}\,\alpha_2}$$

$$\left[\;\cos\alpha_i \;=\; \frac{a}{s_i} \quad \& \quad \sin\alpha_3 \;=\; \frac{h_i}{s_i} \qquad\Rightarrow\qquad S_2 \;=\; \cdots \quad \& \quad S_3 \;=\; \cdots\;\right]$$

primjerice, za $P = 125\,\mathrm{kN}$, $a = 4\,\mathrm{m}$, $h_2 = 2\,\mathrm{m}$ i $h_3 = 3\,\mathrm{m}$ su

$$A \;=\; -\frac{1}{4}\,P \;=\; -\frac{1}{4}\cdot 125 \;=\; -31{,}25\,\mathrm{kN}$$

$$A \;=\; \frac{5}{4}\,P \;=\; \frac{5}{4}\cdot 125 \;=\; 156{,}25\,\mathrm{kN}$$

$$S_1 \;=\; -\frac{a}{2\,h}\,P \;=\; -\frac{4}{2\cdot 5}\cdot 125 \;=\; -50\,\mathrm{kN}$$

$$S_4 \;=\; \frac{a}{2\,h}\,P \;=\; \frac{4}{2\cdot 5}\cdot 125 \;=\; 50\,\mathrm{kN}$$

$$S_2 \;=\; -\frac{\sqrt{a^2 + h_2^2}}{4h}\,P \;=\; -\frac{\sqrt{4^2 + 2^2}}{4\cdot 5}\cdot 125 \;=\; -27{,}950\,8\,\mathrm{kN}$$

$$S_3 \;=\; -\frac{\sqrt{a^2 + h_3^2}}{4h}\,P \;=\; -\frac{\sqrt{4^2 + 3^2}}{4\cdot 5}\cdot 125 \;=\; 31{,}25\,\mathrm{kN}$$

ili:

$$\mathrm{tg}\,\alpha_2 \;=\; \frac{h_2}{a} \;=\; \frac{2}{4} \;=\; 0{,}5 \qquad \& \qquad \mathrm{tg}\,\alpha_3 \;=\; \frac{h_3}{a} \;=\; \frac{3}{4} \;=\; 0{,}75$$

$$\mathrm{tg}\,\alpha_2 \;+\; \mathrm{tg}\,\alpha_3 \;=\; \frac{5}{4} \;=\; 1{,}25$$

$$S_{2,3} \;=\; \frac{1}{4}\,P \;=\; 31{,}25\,\mathrm{kN}$$

$$S_2 \;=\; -\frac{\sqrt{1 \;+\; \mathrm{tg}^2\,\alpha_2}}{\mathrm{tg}\,\alpha_2 \;+\; \mathrm{tg}\,\alpha_3}\,S_{2,3} \;=\; -\frac{\sqrt{1 \;+\; 0{,}5^2}}{0{,}5 \;+\; 0{,}75}\cdot 31{,}25 \;=\; -27{,}950\,8\,\mathrm{kN}$$

$$S_3 \;=\; -\frac{\sqrt{1 \;+\; \mathrm{tg}^2\,\alpha_3}}{\mathrm{tg}\,\alpha_2 \;+\; \mathrm{tg}\,\alpha_3}\,S_{2,3} \;=\; -\frac{\sqrt{1 \;+\; 0{,}75^2}}{0{,}5 \;+\; 0{,}75}\cdot 31{,}25 \;=\; 31{,}25\,\mathrm{kN}$$

ili:

$$\alpha_2 \;=\; \mathrm{arc\,tg}\,\frac{1}{2} \;=\; 0{,}463\,648 \qquad \& \qquad \alpha_3 \;=\; \mathrm{arc\,tg}\,\frac{3}{4} \;=\; 0{,}643\,501$$

$$\cos\alpha_2 \;=\; 0{,}894\,427, \quad \sin\alpha_2 \;=\; 0,447\,214 \qquad \& \qquad \cos\alpha_3 \;=\; 0{,}8, \quad \sin\alpha_3 \;=\; 0{,}6$$

$$S_2 \;=\; -\frac{S_{2,3}}{\sin\alpha_2 \;+\; \cos\alpha_2\;\mathrm{tg}\,\alpha_3} \;=\; -\frac{31{,}25}{0,447\,214 \;+\; 0{,}894\,427\cdot 0{,}75} \;=\; -27{,}950\,8\,\mathrm{kN}$$

$$S_3 \;=\; -\frac{S_{2,3}}{\sin\alpha_3 \;+\; \cos\alpha_3\;\mathrm{tg}\,\alpha_2} \;=\; -\frac{31{,}25}{0{,}6 \;+\; 0{,}8\cdot 0{,}5} \;=\; 31{,}25\,\mathrm{kN}$$