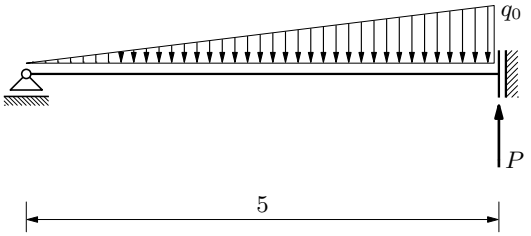
Statički određena greda

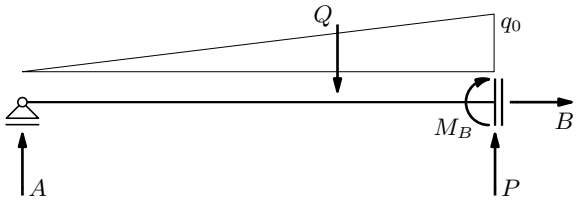
Nacrtajte dijagrame momenata savijanja i poprečnih sila! Izračunajte najveći intenzitet momenta savijanja!

$q_0 \;=\; 50\,\mathrm{kN/m}$

$P \;=\; 100\,\mathrm{kN}$



vrijednosti reakcija:



$$\sum F_x \;=\; 0 \;:\qquad B \;=\; 0$$

$$\sum F_z \;=\; 0 \;:\qquad -A + \underbrace{\frac{1}{2}\,q_0\,\ell}_{Q} - P \;=\; 0$$

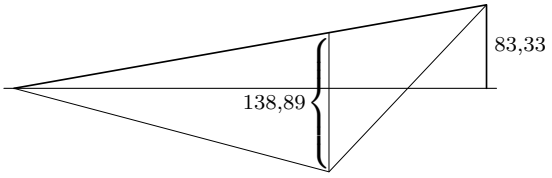
$$Q \;=\; \frac{1}{2}\cdot 50 \cdot 5 \;=\; 125\,\mathrm{kN}$$

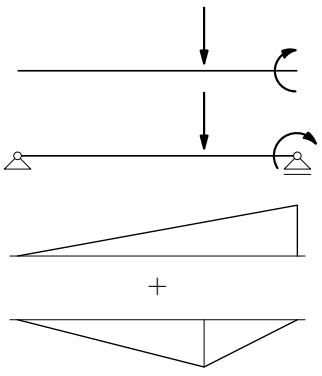
$$A \;=\; Q - P \;=\; 125 - 100 \;=\; 25\,\mathrm{kN}$$

$$\sum M_{/A} \;=\; 0 \;:\qquad -\frac{2}{3}\,\ell\,Q - M_B + \ell\,P \;=\; 0$$

$$M_B \;=\; -\frac{2}{3}\,\ell\,Q + \ell\,P \;=\; -\frac{2}{3}\cdot 5 \cdot 125 + 5 \cdot 100 \;=\; 83{,}333\,3\,\mathrm{kNm}$$

dijagrami:

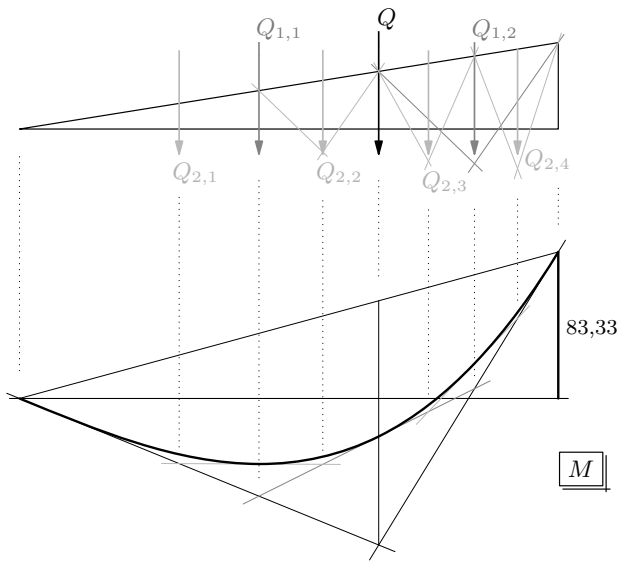


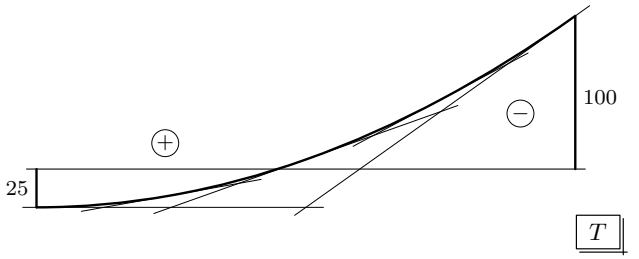


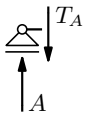
$$\frac{1}{3}\,\ell\,Q - \ell\cdot\bar{A} \;=\; 0 \qquad \Rightarrow \qquad \bar{A} \;=\; \frac{1}{3}\,Q$$

$$\overline{M}_Q \;=\; \frac{2}{3}\,\ell\,\bar{A} \;=\; \frac{2}{9}\,Q\,\ell \;=\; \frac{2}{9}\left(\frac{1}{2}\,q_0\,\ell\right)\ell \;=\; \frac{q_0\,\ell^2}{9}$$

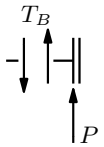
$$=\; \frac{50\cdot 5^2}{9} \;=\; 138{,}889\,\mathrm{kNm}$$





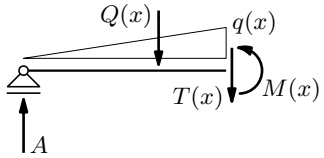


$$-A + T_A = 0$$
$$T_A = A = 25\,\text{kN}$$



$$-T_B - P = 0$$
$$T_B = -P = -100\text{kN}$$

moment najvećega intenziteta:



$$q(x) = \frac{q_0}{\ell}\,x = \frac{50}{5}\,x = 10 \cdot x$$
$$Q(x) = \frac{1}{2}\,q(x)\,x = \frac{1}{2} \cdot (10 \cdot x) \cdot x = 5 \cdot x^2$$

$$\sum M_{/x} = 0: \qquad -x\,A + \frac{1}{3}\,x\,Q(x) + M(x) = 0$$
$$M(x) = x \cdot 25 - \frac{x}{3} \cdot \left(5 \cdot x^2\right) = 25 \cdot x - \frac{5}{3} \cdot x^3$$
$$\text{provjera:}\quad M(\ell) = 25 \cdot 5 - \frac{5}{3} \cdot 5^3 = -83{,}333\,3\,\text{kNm} \quad \circlearrowleft$$
$$\sum F_z = 0: \qquad -A + Q(x) + T(x) = 0$$
$$T(x) = 25 - 5 \cdot x^2$$
$$\text{provjera:}\quad T(0) = 25\,\text{kN} \quad \& \quad T(\ell) = 25 - 5 \cdot 5^2 = -100\,\text{kN}$$
$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} = 25 - 3 \cdot \frac{5}{3} \cdot x^2 = 25 - 5 \cdot x^2 = T(x)$$
$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} = 0 \qquad \left(\text{ili} \quad T(x) = 0\right)$$
$$25 - 5 \cdot x^2 = 0 \qquad \Rightarrow \qquad x^2 = 5 \qquad \Rightarrow \qquad x_{1,2} = \mp\sqrt{5} = \mp 2{,}236\,07\ [\text{m}]$$
$$M_{\max} = M(2{,}236) = 37{,}267\,8\,\text{kNm}$$

varijacija na temu:

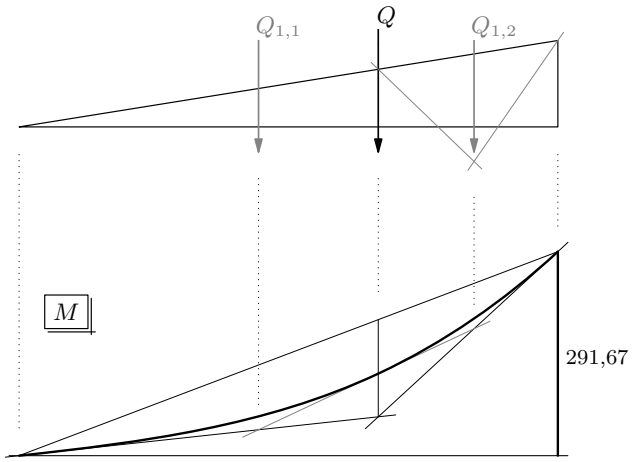
$q_0 \;=\; 25\,\mathrm{kN/m} \qquad\qquad Q \;=\; \dfrac{1}{2}\,q_0\,\ell \;=\; \dfrac{1}{2}\cdot 25\cdot 5 = 62{,}5\,\mathrm{kN}$

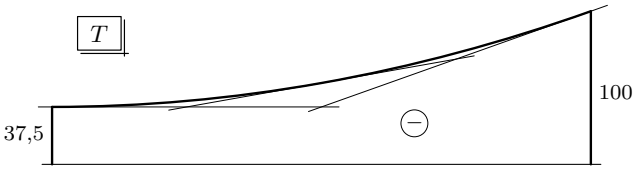
$P \;=\; 100\,\mathrm{kN}$

$A \;=\; Q \;-\; P \;=\; 62{,}5 \;-\; 100 \;=\; -37{,}5\,\mathrm{kN} \quad \downarrow$

$M_B \;=\; -\dfrac{2}{3}\,\ell\,Q \;+\; \ell\,P \;=\; -\dfrac{2}{3}\cdot 5\cdot 62{,}5 \;+\; 5\cdot 100 \;=\; 291{,}667\ \mathrm{kNm}$

$\overline{M}_Q \;=\; \dfrac{q_0\,\ell^2}{9} \;=\; \dfrac{25\cdot 5^2}{9} \;=\; 69{,}444\,4\,\mathrm{kNm}$





$T_A \;=\; A \;=\; -37{,}5\,\mathrm{kN}$

$T_B \;=\; -P \;=\; -100\,\mathrm{kN}$

$q(x) \;=\; \dfrac{25}{5}\,x \;=\; 5\cdot x \qquad\qquad Q(x) \;=\; \dfrac{1}{2}\cdot(5\cdot x)\cdot x \;=\; \dfrac{5}{2}\cdot x^2$

$M(x) \;=\; x\,A \;-\; \dfrac{x}{3}\,Q(x) \;=\; x\cdot(-37{,}5) \;-\; \dfrac{x}{3}\cdot\left(\dfrac{5}{2}\cdot x^2\right) \;=\; -37{,}5\cdot x \;-\; \dfrac{5}{6}\cdot x^3$

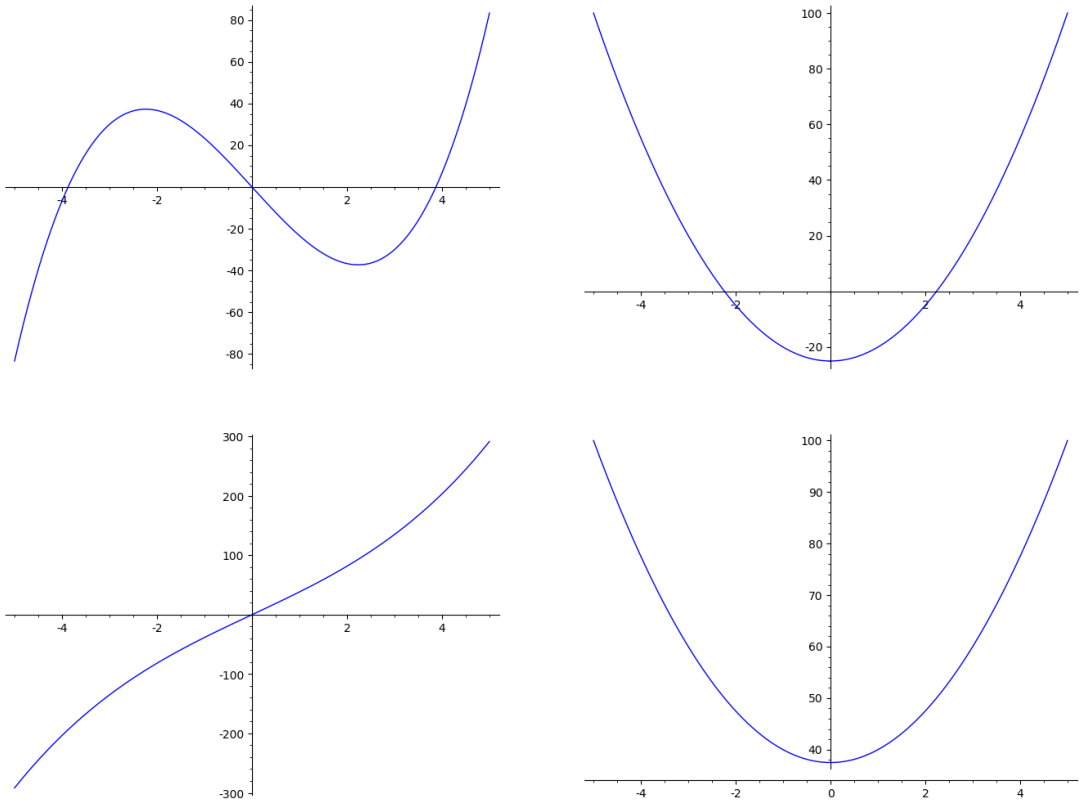
provjera: $M(\ell) \;=\; -37{,}5\cdot 5 \;-\; \dfrac{5}{6}\cdot 5^3 \;=\; -291{,}667\,\mathrm{kNm} \qquad \circlearrowleft$

$T(x) \;=\; A \;-\; Q(x) \;=\; -37{,}5 \;-\; \dfrac{5}{2}\cdot x^2$

provjera: $T(\ell) \;=\; -37{,}5 \;-\; \dfrac{5}{2}\cdot 5^2 \;=\; -100\,\mathrm{kNm}$

$\dfrac{\mathrm{d}M(x)}{\mathrm{d}x} \;=\; -37{,}5 \;-\; 3\cdot\dfrac{5}{6}\cdot x^2 \;=\; -37{,}5 \;-\; \dfrac{5}{2}\cdot x^2 \;=\; T(x)$

$\dfrac{\mathrm{d}M(x)}{\mathrm{d}x} \;=\; 0 \qquad \Rightarrow \qquad x^2 \;=\; \dfrac{2}{5}\,(-37{,}5) \qquad \Rightarrow \qquad x \;=\; \mp\sqrt{-15}$

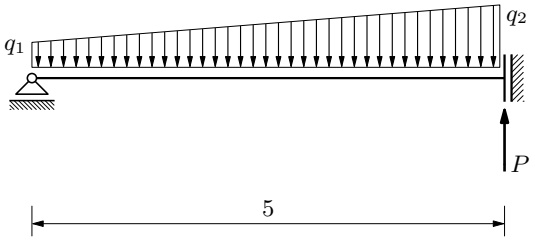


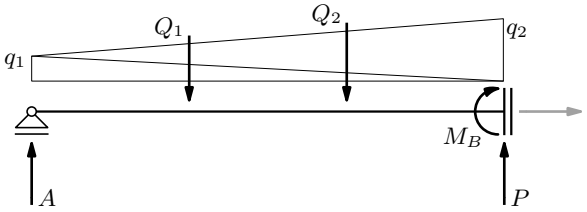
još jedna varijacija:

$q_1 = 10\,\text{kNm}$

$q_2 = 25\,\text{kN/m}$

$P = 100\,\text{kN}$





$$Q_1 = \frac{1}{2}\, q_1\, \ell = \frac{1}{2} \cdot 10 \cdot 5 = 25\,\text{kN}$$

$$Q_2 = \frac{1}{2}\, q_2\, \ell = \frac{1}{2} \cdot 25 \cdot 5 = 62{,}5\,\text{kN}$$

$$\sum F_z = 0: \qquad -A + Q_1 + Q_2 - P = 0$$

$$A = Q_1 + Q_2 - P = = -12{,}5\,\text{kN}$$

$$\sum M_{/A} \;=\; 0 \;: \qquad -\frac{1}{3}\,\ell\,Q_1 \;-\; \frac{2}{3}\,\ell\,Q_2 \;-\; M_B \;+\; \ell\,P \;=\; 0$$

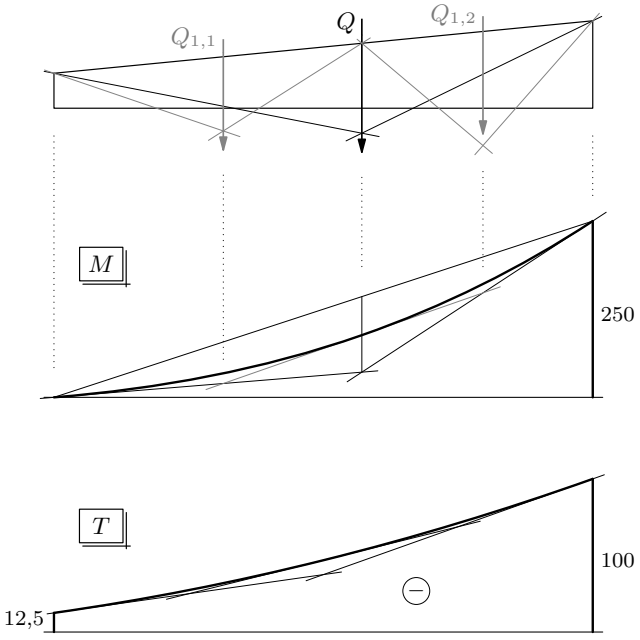
$$M_B \;=\; -\frac{1}{3}\cdot 5\cdot 25 \;-\; \frac{2}{3}\cdot 5\cdot 62{,}5 \;+\; 5\cdot 100 \;=\; 250\,\mathrm{kNm}$$

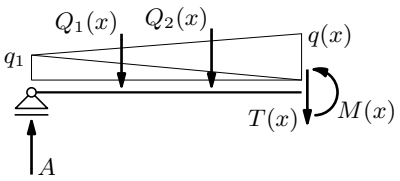
$$Q \;=\; Q_1 \;+\; Q_2 \;=\; 25 \;+\; 62{,}5 \;=\; 87{,}5\,\mathrm{kN}$$

$$x_Q\,Q \;=\; \frac{1}{3}\,\ell\,Q_1 \;+\; \frac{2}{3}\,\ell\,Q_2 \qquad \Rightarrow \qquad x_Q \;=\; \frac{4}{7}\,\ell \;=\; \frac{20}{7} \;=\; 2{,}857\,14\,\mathrm{m}$$

$$-\ell\,\bar{A} \;+\; (\ell - x_Q)\,Q \;=\; 0 \qquad \Rightarrow \qquad \bar{A} \;=\; \left(1 - \frac{4}{7}\right) Q \;=\; \frac{3}{7}\,Q \;=\; 37{,}5\,\mathrm{kN}$$

$$\overline{M}_Q \;=\; x_Q\,\bar{A} \;=\; \frac{12}{49}\,\ell\,Q \;=\; 107{,}143\,\mathrm{kNm}$$





$$q(x) \;=\; q_1 \;+\; \frac{q_2 - q_1}{\ell}\,x \;=\; 10 \;+\; \frac{15}{5}\,x \;=\; 10 \;+\; 3\cdot x$$

$$Q_1(x) \;=\; \frac{1}{2}\cdot 10\cdot x \;=\; 5\cdot x$$

$$Q_2(x) \;=\; \frac{1}{2}\cdot (10 + 3\cdot x)\cdot x \;=\; 5\cdot x \;+\; \frac{3}{2}\cdot x^2$$

$$-x\,A \;+\; \frac{2}{3}\,x\,Q_1(x) \;+\; \frac{1}{3}\,x\,Q_2(x) \;+\; M(x) \;=\; 0$$

$$M(x) \;=\; x\cdot(-12{,}5) \;-\; \frac{2}{3}\cdot x\cdot(5\cdot x) \;-\; \frac{1}{3}\cdot x\cdot\left(5\cdot x \;+\; \frac{3}{2}\cdot x^2\right) \;=\; -12{,}5\cdot x \;-\; 5\cdot x^2 \;-\; \frac{1}{2}\cdot x^3$$

provjera: $\quad M(\ell) \;=\; -12{,}5\cdot 5 \;-\; 5\cdot 5^2 \;-\; \frac{1}{2}\cdot 5^3 \;=\; -250\,\mathrm{kNm} \qquad \circlearrowleft$

$$-A + Q_1(x) + Q_2(x) + T(x) = 0$$

$$T(x) = -12{,}5 - 5 \cdot x - 5 \cdot x - \frac{3}{2} \cdot x^2 = -12{,}5 - 10 \cdot x - \frac{3}{2} \cdot x^2$$

provjera: $T(\ell) = -12{,}5 - 10 \cdot 5 - \frac{3}{2} \cdot 5^2 = -100\,\mathrm{kN}$

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} = -12{,}5 - 2 \cdot 5 \cdot x - 3 \cdot \frac{1}{2} \cdot x^2 = -12{,}5 - 10 \cdot x - \frac{3}{2} \cdot x^2 = T(x)$$

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} = 0 : \qquad -\frac{3}{2} \cdot x^2 - 10 \cdot x - 12{,}5 = 0 \qquad \rightarrow \qquad \frac{3}{2} \cdot x^2 + 10 \cdot x + 12{,}5 = 0$$

$$x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \mp \sqrt{10^2 - 4 \cdot \frac{3}{2} \cdot 12{,}5}}{2 \cdot \frac{3}{2}} \qquad \Rightarrow \qquad x_1 = -5 \quad \& \quad x_2 = -\frac{5}{3}$$