GS 1. — 4. veljače 2025.

Zadatak 3.

a. Za odabrani osnovni sistem izračunajte komponente matrice popustljivosti (koeficijente popustljivosti) zadanoga sistema!

b. Pomoću matrice popustljivosti izračunajte potrebne vrijednosti i nacrtajte dijagrame unutarnjih sila!

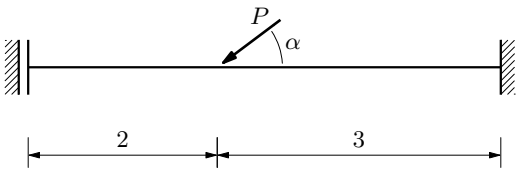
c. Izračunajte orijentiranu duljinu pomaka lijevoga ležaja!

$P = 100\,\text{kN}$

$\text{tg}\,\alpha = {}^{3}/_{4}$

$E = 3 \cdot 10^7\,\text{kN/m}^2$

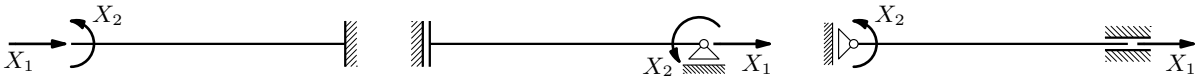
${}^{b}/_{h} = {}^{30}/_{45}\,[\text{cm}]$



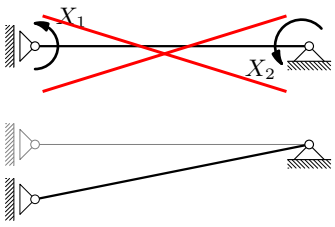
$A = bh = 0{,}3 \cdot 0{,}45 = 0{,}135\,\text{m}^2$ $\qquad EA = 4\,050\,000\,\text{kN}$

$I = \dfrac{bh^3}{12} = \dfrac{0{,}3 \cdot 0{,}45^3}{12} = 0{,}002\,278\,125\,\text{m}^4$ $\qquad EI = 68\,343{,}75\,\text{kNm}^2$

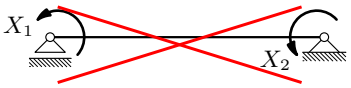
a. mogući/smisleni/ispravni osnovni sistemi:



nemogući „osnovni sistem”:

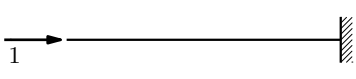


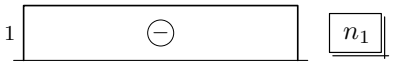
„osnovni sistem” za pad na ispitu:



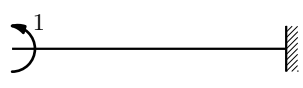
[što je sve ovdje krivo?]

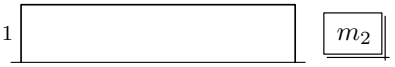
odabrani osnovni sistem: prvi smisleni (konzola)





$m_1 = 0$





$n_2 = 0$

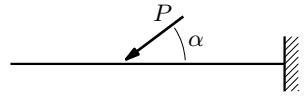
$$\delta_{1,1} \;=\; \int \frac{n_1^2}{EA}\,\mathrm{d}x \;+\; \int \frac{m_1^2}{EI}\,\mathrm{d}x \;=\; \frac{1}{EA}\,(1\cdot\ell)\cdot 1 \;=\; \frac{\ell}{EA} \;=\; \frac{5}{EA}$$

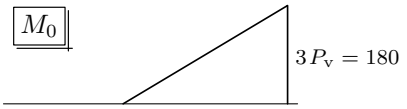
$$\delta_{2,2} \;=\; \int \frac{n_2^2}{EA}\,\mathrm{d}x \;+\; \int \frac{m_2^2}{EI}\,\mathrm{d}x \;=\; \frac{1}{EI}\,(1\cdot\ell)\cdot 1 \;=\; \frac{\ell}{EI} \;=\; \frac{5}{EI}$$

$$\delta_{1,2} \;=\; \delta_{2,1} \;=\; \int \frac{n_1\,n_2}{EA}\,\mathrm{d}x \;+\; \int \frac{m_1\,m_2}{EI}\,\mathrm{d}x \;=\; 0$$

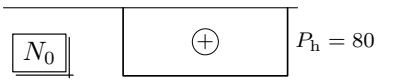
$$\mathbf{D} \;=\; \begin{bmatrix} \delta_{1,1} & \delta_{1,2} \\ \delta_{2,1} & \delta_{2,2} \end{bmatrix} \;=\; \begin{bmatrix} \dfrac{\ell}{EA} & 0 \\ 0 & \dfrac{\ell}{EI} \end{bmatrix} \;=\; \begin{bmatrix} \dfrac{5}{EA} & 0 \\ 0 & \dfrac{5}{EI} \end{bmatrix} \;=\; \begin{bmatrix} 1{,}234\,57\cdot 10^{-6} & 0 \\ 0 & 7315\,96\cdot 10^{-5} \end{bmatrix}$$

b. $\operatorname{tg}\,\alpha \;=\; \dfrac{3}{4} \qquad \Rightarrow \qquad P_\mathrm{v} \;=\; \dfrac{3}{5}\,P \;=\; 60\,\mathrm{kN} \qquad \& \qquad P_\mathrm{h} \;=\; \dfrac{4}{5}\,P \;=\; 80\,\mathrm{kN}$





$$\delta_{1,0} \;=\; \int \frac{N_0\,n_1}{EA}\,\mathrm{d}x \;=\; \frac{1}{EA}\,\big(P_\mathrm{h}\cdot 3\big)\cdot 1\cdot(-1) \;=\; -\frac{240}{EA}$$



$$\delta_{2,0} \;=\; \int \frac{M_0\,m_2}{EI}\,\mathrm{d}x \;=\; \frac{1}{EI}\left(\frac{1}{2}\cdot 3\,P_\mathrm{v}\cdot 3\right)\cdot 1 \;=\; \frac{270}{EI}$$

formalno (kao što se u zadatku traži):

$$\mathbf{D}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \delta_{1,0} \\ \delta_{2,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \mathbf{D}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\begin{bmatrix} \delta_{1,0} \\ \delta_{2,0} \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\mathbf{D}^{-1}\begin{bmatrix} \delta_{1,0} \\ \delta_{2,0} \end{bmatrix}$$

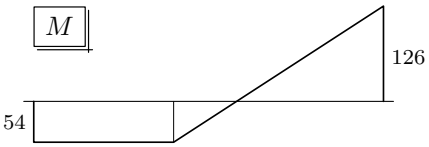
$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \;=\; -\begin{bmatrix} \dfrac{EA}{5} & 0 \\ 0 & \dfrac{EI}{5} \end{bmatrix}\begin{bmatrix} -\dfrac{240}{EA} \\ \dfrac{270}{EI} \end{bmatrix} \;=\; \begin{bmatrix} 48 \\ -54 \end{bmatrix}$$

ili jednostavno

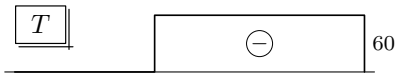
$$\delta_{1,1}\,X_1 \;=\; -\delta_{1,0} \qquad \Rightarrow \qquad X_1 \;=\; -\frac{\delta_{1,0}}{\delta_{1,1}} \;=\; -\frac{-\frac{240}{EA}}{\frac{5}{EA}} \;=\; 48$$

$$\delta_{2,2}\,X_2 \;=\; -\delta_{2,0} \qquad \Rightarrow \qquad X_2 \;=\; -\frac{\delta_{2,0}}{\delta_{2,2}} \;=\; -\frac{\frac{270}{EI}}{\frac{5}{EI}} \;=\; -54$$

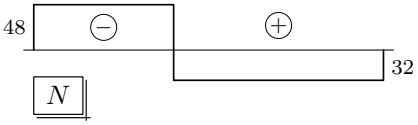
$$M(x) \;=\; M_0(x) \;+\; X_2\, m_2(x)$$



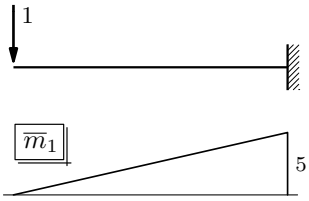
$$T(x) \;=\; M'(x)$$

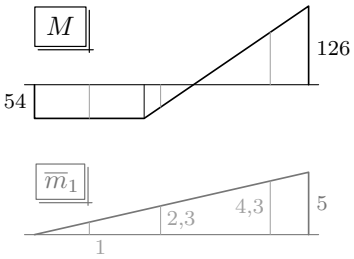


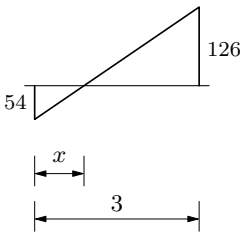
$$N(x) \;=\; N_0(x) \;+\; X_1\, n_1(x)$$



c. (primjena redukcijskoga „stavka” [koji glasi. . . ?])







$$\frac{x}{54} \;=\; \frac{3}{54+126}$$

$$x \;=\; \frac{3\cdot 54}{180} \;=\; 0{,}9$$

$$w \;=\; \int \frac{M\,\overline{m}_1}{EI}$$

$$\;=\; \frac{1}{68\,343{,}75}\left[(54\cdot 2)\cdot 1\cdot(-1) + \left(\frac{1}{2}\cdot 54\cdot 0{,}9\right)\cdot 2{,}3\cdot(-1) + \left(\frac{1}{2}\cdot 126\cdot 2{,}1\right)\cdot 4{,}3\right]$$

$$\;=\; 0{,}005\,925\,93\ \mathrm{m} \;=\; 5{,}9\ \mathrm{mm}$$

ili, malo jednostavnije:

$$w \;=\; \frac{1}{68\,343{,}75}\left[(54\cdot 5)\cdot 2.5\cdot(-1) + \left(\frac{1}{2}\cdot 180\cdot 3\right)\cdot 4\right]$$

$$\;=\; 0{,}005\,925\,93\ \mathrm{m}$$