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Geometric Modelling of Hyperpatches

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ABSTRACT
The paper deals with the modelling of solids (hyperpatches) on the basis of their creative laws. Creative representation of a solid enables to modell also solids with "curve-like" edges, not only solids with the polyhedral boundary as in the boundary representation method. There is provided also the possibility to control a non-homogeneous distribution of the interior points of a solid created as an interpolated figure. Basic notions such as a solid cell, an isoparametric curve segment and an isoparametric surface patch, or tangent space and density vector in a solid point are described and their relevance to the intrinsic geometric properties of the solid is discussed. Composite solid modelling problems on adjoining of the elementar solid cells are mentioned.
Keywords: solid modelling, creative space, interpolation of hyperpatches

Geometrijsko modeliranje po dijelovima hiperplohama
SAŽETAK
Rad se bavi modeliranjem tijela (solids) na osnovi njihovih zakona stvaranja (creative laws). Kreativno predstavljanje tijela omogućava i modeliranje tijela koja imaju "zakrivljene" stranice, ne samo tijela s poliedarskim stranicama kao što je to slučaj u metodi prezentiranja rubova (boundary representation method). Postoji također i mogućnost upravljanja nehomogenom raspodjelom unutarnjih točaka tijela kreiranog poput interpoliranog lika. Opisani su osnovni pojmovi kao što su ćelija tijela, segment izoparametarske krivulje i dio izoparametarske plohe, ili tangentni prostor i vektor gustoće u točki tijela, a raspravlja se i o njihovoj primjerenosti unutarnjim geometrijskim osobinama tijela. Spomenuti su problemi modeliranja složenog tijela pri dodavanju elementarnih ćelija.
Ključne riječi: modeliranje tijela, kreativni prostor, interpolacija po dijelovima hiperplohama

1. INTRODUCTION

Solid modelling is one of those interesting problems, which are still developing parts of Computer Graphics. Solids can be created due to their creative laws or by description of the incidence structure of their boundary. There exists a lot of literature concerned the second mentioned way of modelling, this topic is given in details in Mantyla [1].

Let us look therefore in details at the modelling of solids on the basis of their creative laws and representations in the Creative space.

Let K be a Creative space, an ordered pair $K = (U,G)$, where U - base is a set of figures in the space (subsets of the extended Euclidean space $\bar{E}_3$) and G- generator is a set of generating principles ($G = GP(\bar{E}_3) \cup L$, while $GP(\bar{E}_3)$ is a group of projective transformations in $\bar{E}_3$ and L is a set of interpolations). More detailed description can be found in Velichovà [3], [6].

Solid T (a three-parametric subset of $\bar{E}_3$) is in K synthetically represented by its creative representation, an ordered pair (U,G), where U $\in$ U (a basic figure) and G $\in$ G (a generating principle) are such, that applying the generating principle G on the basic figure U the created solid T will be obtained. Generating principle G can be a geometric transformation, a class of geometric transformations or any interpolation. The first two types of generating principles provide modelling of solids which are homogeneous, i.e. their interior points are uniformly distributed. This feature of the uniform points' distribution is implicitly assumed in the case of solids defined by their incidence structure describing order and incidence of all elements (vertices, edges, faces) of the boundary of solids as three-dimensional regions in $\bar{E}_3$. In this case we even restrict our considerations on polyhedra only, while using creative representation we can create also more complex shaped solids with "curve-like" edges.

The possibility to describe and to control the feature of the non-uniform distribution of points in a solid is provided in the case of its modelling as an interpolated figure, using its creative representation in which the generating principle is an interpolation. This fact can play a very important role in many technical branches (science of materials, timber industry) but also in the sphere of medical diagnostics and image processing. Industrial design and CAGD of non-homogeneous solids using new efficient computer systems seem to be very perspective.

2. BASIC NOTIONS

Elementary notions and considerations concerned the problem of the solid interpolation can be easily deduced by analogy with the interpolation of curves and surfaces, increasing the dimension of a figure to three.

Analytic representation of a hyperpatch - solid cell C is a vector function

$$\mathbf{r}(u,v,w) = (x(u,v,w), y(u,v,w), z(u,v,w), h(u,v,w))$$

defined on the region $\Omega = [0,1]^3$ (where x, y, z and h are homogeneous coordinate functions at least C^3 continuous on a given region Ω), which is a local homeomorphic mapping of the region Ω on the hyperpatch C (according to Velichová [5]). A notion hyperpatch - cell of a solid analogously corresponds to notion of a curve segment or a surface patch. Composite solid can be obtained as a composition of several elementary cells. A notion isoparametric surface of a solid can be coordinated to a notion isoparametric curve of a surface. There exist three isoparametric systems of surfaces (exactly one of parameters u, v, w is constant) forming a net of surfaces in a solid. Boundary surfaces (faces) correspond to the constant values equal to 0 or 1. Setting two of parameters u, v, w equal to a constant value we can speak about isoparametric curves in a solid, and if the values are equal to 0 or 1 about boundary isoparametric curves (edges). Two isoparametric surfaces from different systems intersect in an isoparametric curve, two isoparametric curves intersect in a point. In a point of a solid all three parameters are constant and we denote them as parametric (curvilinear) coordinates of a solid point. Points with parametric coordinates equal to 0 or 1 only are vertices of a solid. A hyperpatch boundary (see Fig. 1) consists of

6 boundary surface patches - face surfaces of a hyperpatch

$\mathbf{r}(0,v,w)$, $\mathbf{r}(1,v,w)$, $\mathbf{r}(u,0,w)$, $\mathbf{r}(u,1,w)$, $\mathbf{r}(u,v,0)$, $\mathbf{r}(u,v,1)$

12 boundary curve segments - edge curves of a hyperpatch

$\mathbf{r}(0,0,w)$, $\mathbf{r}(0,1,w)$, $\mathbf{r}(1,0,w)$, $\mathbf{r}(1,1,w)$,
$\mathbf{r}(0,v,0)$, $\mathbf{r}(0,v,1)$, $\mathbf{r}(1,v,0)$, $\mathbf{r}(1,v,1)$,
$\mathbf{r}(u,0,0)$, $\mathbf{r}(u,0,1)$, $\mathbf{r}(u,1,0)$, $\mathbf{r}(u,1,1)$,

8 corner points - vertices of a hyperpatch

$\mathbf{r}(0,0,0)$, $\mathbf{r}(0,1,0)$, $\mathbf{r}(1,0,0)$, $\mathbf{r}(1,1,0)$,
$\mathbf{r}(0,0,1)$, $\mathbf{r}(0,1,1)$, $\mathbf{r}(1,0,1)$, $\mathbf{r}(1,1,1)$.

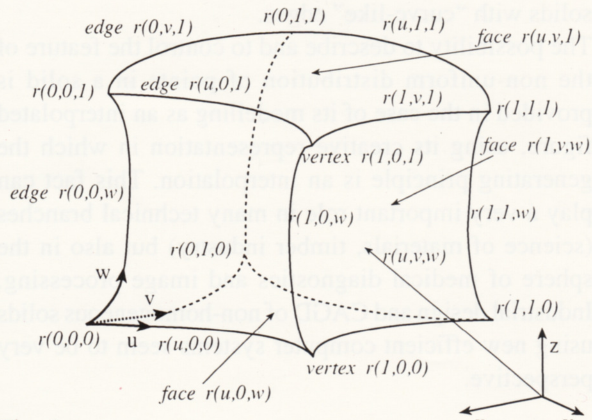

Fig. 1

The analytic representation of a hyperpatch can be obtained easily, from the creative representation, by the multiplication of matrices, which are analytic representations of the creative representation elements. Creative representation of a hyperpatch, an ordered pair (U,G), can have six possible forms according to the different types of the basic figure U and the suitable generating principle G:

(a surface patch, a class of geometric transformations)
(a solid cell, a geometric transformation)
(a net of boundary surface patches, an interpolation)
(a sequence of surface patches, an interpolation)
(a grid of boundary edge curves, an interpolation)
(a grid of points, an interpolation).

In the first two types of the hyperpatch creative representation the analytic representation of a generating principle is a regular square matrix of rank 4 with real numbers as elements (for a geometric transformation) or elements in a form of the real functions of one real variable all defined and at least C^1 continuous at the unit closed interval [0,1] (for a class of geometric transformations). In both cases the basic figure is analytically represented by a vector function, of one variable for a curve segment and of two variables for a surface patch, defined and satisfying certain conditions on the closed regions, as described in Velichová [3], [5]. Hyperpatches determined by these representations are homogeneous, and different types of created solids are described in Velichová [4]. For example, a solid of revolution can be created from a surface patch subdued to a class of revolutions about an axis, or a ruled solid from the same basic figure subdued to a class of translations. In the Fig. 2 there are illustrations of several solids generated by classes of geometric transformations.

Next 3 types of creative representations use as the basic figures ordered sets of points, curve segments or surface patches. These can be analytically represented by their vector equations and form the elements of the matrices - analytic representation of the basic figures - maps of the created hyperpatches, distributed in the appropriate order. Generating principle is in all 3 types an interpolation.

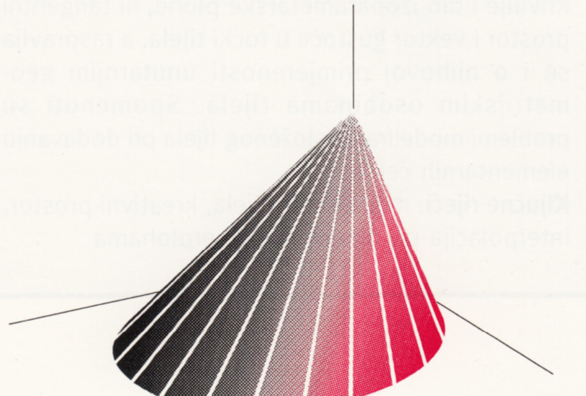

Fig. 2a

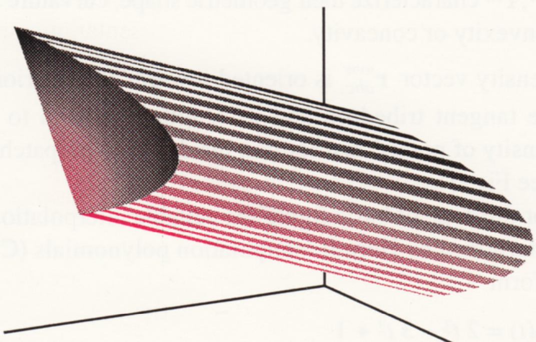

Fig. 2b

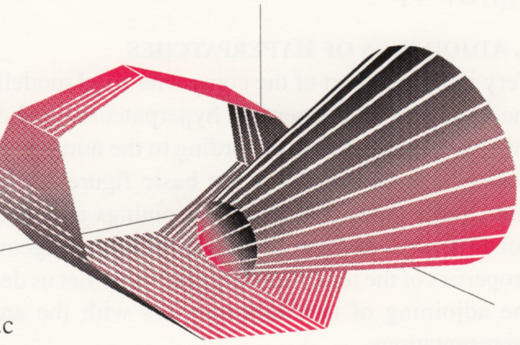

Fig. 2c

3. Interpolation of Solids

In the following, we will describe the tri-cubic interpolation (cubic interpolation in three different directions) of a hyperpatch. Analytic representation of a tri-cubic solid cell is in a form

$$\mathbf{r}(u,v,w) = \mathbf{a}_{333}u^3v^3w^3 + \mathbf{a}_{332}u^3v^3w^2 + \ldots + \mathbf{a}_{100}u + \mathbf{a}_{000} = $$
$$= \sum_{i=0}^{3}\sum_{j=0}^{3}\sum_{k=0}^{3} \mathbf{a}_{ijk}\, u^i v^j w^k$$
$$= \sum_{i=0}^{3}\sum_{j=0}^{3}\sum_{k=0}^{3} F_i(u)\, F_j(v)\, F_k(w)\, \mathbf{b}_{ijk}$$

for $(u,v,w) \in [0,1]^3$, where $F_i(u)$, $F_j(v)$, $F_k(w)$ are cubic interpolation polynomials.

The set up of algebraic coefficients $\mathbf{a}_{ijk}$ (64 vectors) does not reveal clearly geometric features of the interpolated hyperpatch, while the set up of geometric coefficients $\mathbf{b}_{ijk}$ defines directly geometric properties of a hyperpatch and these even form a basic figure in the hyperpatch creative representation. From the analytic point of view geometric coefficients $\mathbf{b}_{ijk}$ form a three-dimensional matrix of the type 4×4×4, the map of a hyperpatch. The elements of this map are analytic representations of the hyperpatch basic figure elements, it means quadruples of homogeneous coordinates of the hyperpatch points (finite points in $\overline{E}_3$), tangent vectors of the hyperpatch edges, twist vectors of the hyperpatch faces, and vectors defining the distribution of points inside the hyperpatch - density vectors (points in $\overline{E}_3$ at infinity).

Let the basic figure of a hyperpatch be an ordered grid of 64 finite points in $\overline{E}_3$. According to the type of the used interpolation polynomials we can obtain a tri-cubic interpolation hyperpatch containing all elements of its basic figure (polynomials (A)), or such, that contains only 8 points of the given basic grid (Bernstein cubic polynomials (B)). In the first type, the basic grid of points defines also the curvature of edges and faces, and density of the hyperpatch, i.e. distribution of points inside. Analogy of a Beziér patch, approximation Beziér cubic hyperpatch (cell) defined by a grid of 4×4×4 points consists of faces, which are Beziér cubic approximation patches. Edges are Beziér cubic curve segments passing through the 8 corner points. Density of points' distribution inside the cell is an approximation of the order and position of points inside the basic figure grid and it is defined implicitly in the basic figure of the hyperpatch.

Interpolation polynomials are for $t = u, v, w$ in a form

$$F_0(t) = -4.5\,t^3 + 9\,t^2 - 5.5\,t + 1$$
$$F_1(t) = 13.5\,t^3 - 22.5\,t^2 + 9\,t$$
$$F_2(t) = -13.5\,t^3 + 18\,t^2 - 4.5\,t$$
$$F_3(t) = 4.5\,t^3 - 4.5\,t^2 + t \qquad \text{(A)}$$

$$F_0(t) = (1-t)^3$$
$$F_1(t) = 3\,t\,(1-t)^2$$
$$F_2(t) = 3\,t^2\,(1-t)$$
$$F_3(t) = t^3 \qquad \text{(B)}$$

Map of a hyperpatch can be considered as an ordered quadruple of the square matrices $B_k = (P_{ijk})$, $i, j, k = 0,1,2,3$ of the type 4×4. Analytic representation of a hyperpatch is then in a form

$$\mathbf{r}(u,v,w) = \sum_{k=0}^{3} F_k(w) \left(\sum_{i=0}^{3}\sum_{j=0}^{3} F_i(u)\, B_k\, F_j(v) \right),$$
$$(u,v,w) \in [0,1]^3.$$

Let us now consider another basic figure of a hyperpatch, while the elements of this ordered set of points are not only finite points, but also points in $\overline{E}_3$ at infinity - vectors. These describe geometric properties of the created hyperpatch explicitly. The map of such basic figure can be also considered as an ordered quadruple of the square matrices of the rank 4 - arrays B_k, for $k = 0,1,2,3$. Each array is a basic figure of an isoparametric patch and contains also density vectors of the points' distribution in the hyperpatch. Let us establish the following designations (in the Fig.3 illustrated for the curvilinear coordinate values $u = v = w = 0$):

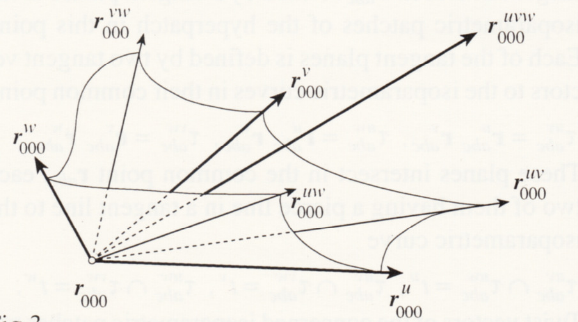

Fig.3

$\mathbf{r}(u,v,w)=\mathbf{r}_{uvw}$ solid point

$\dfrac{\partial \mathbf{r}(u,v,w)}{\partial u}=\mathbf{r}^{u}_{uvw}$ tangent vector to the isoparametric curve

$\dfrac{\partial^2 \mathbf{r}(u,v,w)}{\partial u \partial v}=\mathbf{r}^{uv}_{uvw}$ twist vector to the isoparametric patch

$\dfrac{\partial^3 \mathbf{r}(u,v,w)}{\partial u \partial v \partial w}=\mathbf{r}^{uvw}_{uvw}$ density vector

Separate arrays are then in a form

$$B_0=\begin{pmatrix} \mathbf{r}_{000} & \mathbf{r}_{010} & \mathbf{r}^{v}_{000} & \mathbf{r}^{v}_{010} \\ \mathbf{r}_{100} & \mathbf{r}_{110} & \mathbf{r}^{v}_{100} & \mathbf{r}^{v}_{110} \\ \mathbf{r}^{u}_{000} & \mathbf{r}^{u}_{010} & \mathbf{r}^{uv}_{000} & \mathbf{r}^{uv}_{010} \\ \mathbf{r}^{u}_{100} & \mathbf{r}^{u}_{110} & \mathbf{r}^{uv}_{100} & \mathbf{r}^{uv}_{110} \end{pmatrix}$$

$$B_1=\begin{pmatrix} \mathbf{r}_{001} & \mathbf{r}_{011} & \mathbf{r}^{v}_{001} & \mathbf{r}^{v}_{011} \\ \mathbf{r}_{101} & \mathbf{r}_{111} & \mathbf{r}^{v}_{101} & \mathbf{r}^{v}_{111} \\ \mathbf{r}^{u}_{001} & \mathbf{r}^{u}_{011} & \mathbf{r}^{uv}_{001} & \mathbf{r}^{uv}_{011} \\ \mathbf{r}^{u}_{101} & \mathbf{r}^{u}_{111} & \mathbf{r}^{uv}_{101} & \mathbf{r}^{uv}_{111} \end{pmatrix}$$

$$B_2=\begin{pmatrix} \mathbf{r}^{w}_{000} & \mathbf{r}^{w}_{010} & \mathbf{r}^{vw}_{000} & \mathbf{r}^{vw}_{010} \\ \mathbf{r}^{w}_{100} & \mathbf{r}^{w}_{110} & \mathbf{r}^{vw}_{100} & \mathbf{r}^{vw}_{110} \\ \mathbf{r}^{uw}_{000} & \mathbf{r}^{uw}_{010} & \mathbf{r}^{uvw}_{000} & \mathbf{r}^{uvw}_{010} \\ \mathbf{r}^{uw}_{100} & \mathbf{r}^{uw}_{110} & \mathbf{r}^{uvw}_{100} & \mathbf{r}^{uvw}_{110} \end{pmatrix}$$

$$B_3=\begin{pmatrix} \mathbf{r}^{w}_{001} & \mathbf{r}^{w}_{011} & \mathbf{r}^{vw}_{001} & \mathbf{r}^{vw}_{011} \\ \mathbf{r}^{w}_{101} & \mathbf{r}^{w}_{111} & \mathbf{r}^{vw}_{101} & \mathbf{r}^{vw}_{111} \\ \mathbf{r}^{uw}_{001} & \mathbf{r}^{uw}_{011} & \mathbf{r}^{uvw}_{001} & \mathbf{r}^{uvw}_{011} \\ \mathbf{r}^{uw}_{101} & \mathbf{r}^{uw}_{111} & \mathbf{r}^{uvw}_{101} & \mathbf{r}^{uvw}_{111} \end{pmatrix}$$

Geometric coefficients of an analytic representation of the interpolated hyperpatch are in this case: 8 quadruples of the hyperpatch vertices coordinates, 24 tangent vectors to the hyperpatch edges, 24 twist vectors of the hyperpatch faces and 8 density vectors in the hyperpatch vertices. In this way we describe geometrically not only the boundary of the hyperpatch as a three-dimensional region in the extended Euclidean space $\overline{E}_3$, but also the intrinsic distribution of the region's points.

At any point $\mathbf{r}_{abc}$ of a hyperpatch there is defined a tangent trihedron Γ_{abc} formed by 3 tangent planes to the isoparametric patches of the hyperpatch in this point. Each of the tangent planes is defined by two tangent vectors to the isoparametric curves in their common point,

$$\tau^{uv}_{abc}=\mathbf{r}^{u}_{abc}\ \mathbf{r}^{v}_{abc},\quad \tau^{uw}_{abc}=\mathbf{r}^{u}_{abc}\ \mathbf{r}^{w}_{abc},\quad \tau^{vw}_{abc}=\mathbf{r}^{v}_{abc}\ \mathbf{r}^{w}_{abc}.$$

These planes intersect in the common point $\mathbf{r}_{abc}$, each two of them having a pierce line in a tangent line to the isoparametric curve

$$\tau^{uv}_{abc}\cap\tau^{uw}_{abc}=t^{u},\quad \tau^{uv}_{abc}\cap\tau^{vw}_{abc}=t^{v},\quad \tau^{uw}_{abc}\cap\tau^{vw}_{abc}=t^{w}.$$

Twist vectors of the concerned isoparametric patches $\mathbf{r}^{uv}$, $\mathbf{r}^{uw}$, $\mathbf{r}^{vw}$ characterize their geometric shape, curvature and convexity or concavity.

Density vector $\mathbf{r}^{uvw}_{abc}$ is oriented towards the interior of the tangent trihedron and its length is related to the density of points' distribution inside the hyperpatch (see Fig. 3).

For this type of basic figure the suitable interpolation is determined by Hermit interpolation polynomials (C) in a form

$$F_0(t)=2\,t^3-3\,t^2+1$$
$$F_1(t)=-2\,t^3+3\,t^2$$
$$F_2(t)=t^3-2\,t^2+t$$
$$F_3(t)=t^3-t^2 \qquad \text{(C)}$$

4. ADJOININGS OF HYPERPATCHES

Very important part of the composite solid modelling is the adjoining of elementary hyperpatches - solid cells into a composite solid. According to the number and the type of equal elements in the basic figures of joining cells, three types of different adjoinings can be distinguished, each of them determining different geometric properties of the resulted composite solid. Let us describe the adjoining of two hyperpatches with the analytic representations

$\mathbf{p}(u,v,w)$, $\mathbf{q}(u,v,w)$, $(u,v,w)\in\Omega$,

while the analytic representation of the resulting solid will be denoted as $\mathbf{r}(u,v,w)$.

A. *Continuous adjoining* - G^0 continuity

The two joining cells have a common boundary face patch, it means, in the maps of the joining cells there exist 16 equal (or collinear) elements: 4 quadruples of coordinate vectors of the adjoining boundary patches' vertices, 8 tangent vectors to the edges of the adjoining boundary patches in the corner points and 4 twist vectors of the adjoining boundary patches in these vertices (Fig. 4).

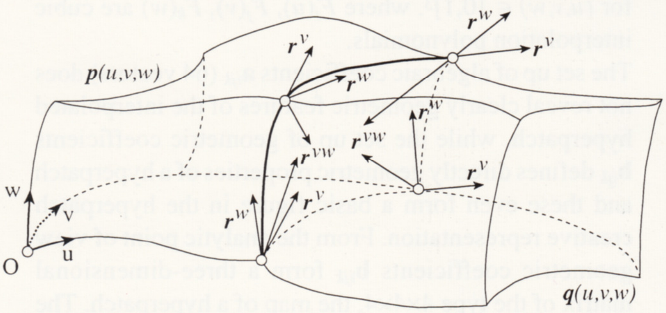

Fig. 4

The composite solid is a continuous region in the extended Euclidean space $\overline{E}_3$ and contains no wholes or bubbles. The adjoining face patches are equal and fit perfectly together, coincide. Isoparametric curve segments which are not parts of the adjoining faces need not be smooth. There can be created new edges in the boundary curve segments of the joining faces and new vertices in their corner points.

$\mathbf{p}(1,v,w)=\mathbf{q}(0,v,w)=\mathbf{r}(0.5,v,w)$

The presented equation expresses in the compound form 16 equations, which determine the collinearity of the vectors representing coinciding faces of the adjoining hyperpatches.

B. ***Smooth adjoining*** - G^1 continuity

Except of the common adjoining face patch, two joining cells must satisfy also the condition of the collinear tangent vectors to all adjoining isoparametric curve segments in the points of the common isoparametric patch. This means, that the adjoining of all isoparametric curve segments not inciting with the common isoparametric patch and 4 adjoining boundary patches is smooth. These conditions can be expressed by the following three compound equations

$$\mathbf{r}^{u}_{0.5vw} = \alpha . \mathbf{p}^{u}_{1vw} = \beta . \mathbf{q}^{u}_{0vw}$$

$$\mathbf{r}^{uw}_{0.5vw} = \gamma . \mathbf{p}^{uw}_{1vw} = \delta . \mathbf{q}^{uw}_{0vw}$$

$$\mathbf{r}^{uv}_{0.5vw} = \varepsilon . \mathbf{p}^{uv}_{1vw} = \varphi . \mathbf{q}^{uv}_{0vw}$$

where α, β, γ, δ, ε, φ are non zero real numbers.

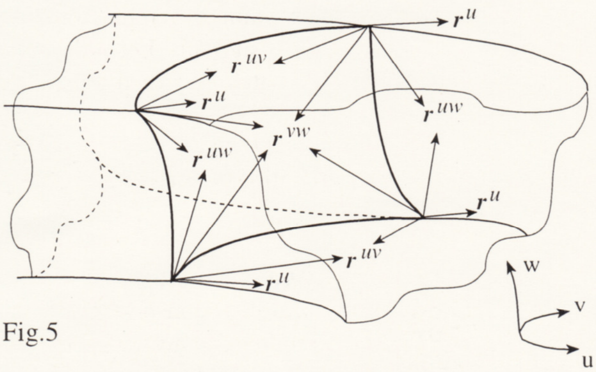

Fig.5

To the former 16 equal (collinear) elements necessary for G^0 continuity we must add 4 tangent vectors to the adjoining edges and 8 twist vectors of the smoothly adjoining boundary patches, two of them in the each of the 4 common vertices. The maps of the joining cells will consist of 28 equal (or collinear) elements (Fig. 5).

C. ***Homogeneous adjoining*** - G^2 continuity

In addition to the equality of all previous geometric parameters of the adjoining cells for G^0 and G^1 continuity, the continuous distribution of the cell's points will be assured as well, requiring the collinearity of density vectors in the common vertices; the maps of the joining cells will have equal (or collinear) 32 elements.

5. APPLICATION POSSIBILITIES

Global properties of the solid models - volume, surface area, moment of inertia, center of gravity, and so on, can be calculated by the evaluating the triple integral

$$\Phi = \int_{Solid} f(\mathbf{r}) dV$$

where Φ is the property required, $f(\mathbf{r})$ is a vector function describing Φ and integration is over the entire volume of the solid.

For a composite solid S created by adjoining of a finite number of cells C_i

$$S = \bigcup_{i=1}^{n} C_i$$

any integral taken over the solid decomposes into the sum of integrals

$$\int_S f(\mathbf{r})\, dV = \sum_{i=1}^{n} \int_{C_i} f(\mathbf{r})\, dV$$

where the cells C_i have disjoint interiors. The methods of evaluating triple integrals on computers are discussed in details by Mortenson [2].

The volume of a solid cell represented analytically by a vector function

$\mathbf{r}(u,v,w) = (\mathbf{x}(u,v,w), \mathbf{y}(u,v,w), \mathbf{z}(u,v,w), \mathbf{h}(u,v,w))$

defined and at least C^3 continuous on the region $\Omega = [0,1]^3$, is the value of the triple integral

$$V = \iiint_{\Omega} (\mathbf{r}^u \mathbf{r}^v \mathbf{r}^w)\, du\, dv\, dw$$

where $(\mathbf{r}^u \mathbf{r}^v \mathbf{r}^w)$ is the triple scalar product of the partial derivatives of the analytic representation - vector function $\mathbf{r}(u,v,w)$ with respect to the variables u, v, w.

Outlined problematic concerned interpolation of solids is a new but very interesting and perspective sphere of a further developement in geometric modelling of figures in E_3. It will undoubtedly serve as a source of a wide field for study of the three-dimensional figures in complexity, it means not excluding their interior density - the distribution of their intrinsic points, which is up till now a sphere ignored by Geometry.

There had been developed a system of easy separate programmes (in QB45 and TPASCAL programming lan-guages) providing calculations and visualizations of solids on the base of the mentioned theory which are used at the Department of Mathematics, Slovak Technical University in Bratislava. Some of them are used regularly in the pedagogical process.

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