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Mollweide Map Projection

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ABSTRACT

Karl Brandan Mollweide (1774-1825) was German mathematician and astronomer. The formulas known after him as Mollweide's formulas are shown in the paper, as well as the proof "without words". Then, the Mollweide map projection is defined and formulas derived in different ways to show several possibilities that lead to the same result. A generalization of Mollweide projection is derived enabling to obtain a pseudocylindrical equal-area projection having the overall shape of an ellipse with any prescribed ratio of its semiaxes. The inverse equations of Mollweide projection has been derived, as well.

The most important part in research of any map projection is distortion distribution. That means that the paper continues with the formulas and images enabling us to get some filling about the linear and angular distortion of the Mollweide projection.

Finally, several applications of Mollweide projections are represented, with the International Cartographic Association logo as an example of one of its successful applications.

Key words: Mollweide, Mollweide's formula, Mollweide map projection

MSC 2010: 51N20, 01A55, 51-03, 51P05, 86A30

Mollweideova kartografska projekcija

SAŽETAK

Karl Brandan Mollweide (1774-1825) bio je njemački matematičar i astronom. U ovom radu prikazane su formule nazvane po njemu kao Mollweideove formule, a uz njih "dokaz bez riječi". Zatim je definirana Mollweideova kartografska projekcija uz izvod formula na nekoliko različitih načina kako bi se pokazalo da postoji više mogućnosti koje vode do istoga rezultata. Izvedena je generalizacija Mollweideove projekcije koja omogućava dobivanje pseudocilindričnih ekvivalentnih (istopovršinskih) projekcija smještenih u elipsu s bilo kojim unaprijed zadanim odnosnom njezinih poluosi. Izvedene su i inverzne jednadžbe Mollweideove projekcije.

Najvažniji dio istraživanja svake kartografske projekcije je ustanovljavanje razdiobe deformacija. Stoga su u radu dane formule i grafički prikazi koji daju uvid u razdiobu linearnih i kutnih deformacija Mollweideove projekcije.

Na kraju je prikazano nekoliko primjena Mollweideove projekcije. Među njima je i logotip Međunarodnoga kartografskog društva, kao jedan od primjera njezine uspješne primjene.

Ključne riječi: Mollweide, Mollweideova formula, Mollweideova kartografska projekcija

1 Mollweide's Formulas

In trigonometry, Mollweide's formula, sometimes referred to in older texts as Mollweide's equations, named after Karl Mollweide, is a set of two relationships between sides and angles in a triangle. It can be used to check solutions of triangles.

Let a, b, and c be the lengths of the three sides of a triangle. Let α, β, and γ be the measures of the angles opposite those three sides respectively. Mollweide's formulas state that

$$\frac{a+b}{c}=\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}} \quad \text{and} \quad \frac{a-b}{c}=\frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\gamma}{2}}.$$

Each of these identities uses all six parts of the triangle - the three angles and the lengths of the three sides.

These trigonometric identities appear in Mollweide's paper *Zusätze zur ebenen und sphärischen Trigonometrie* (1808). A proof without words of these identities (see Fig. 1) is given in DeKleine (1988) and Nelsen (1993).

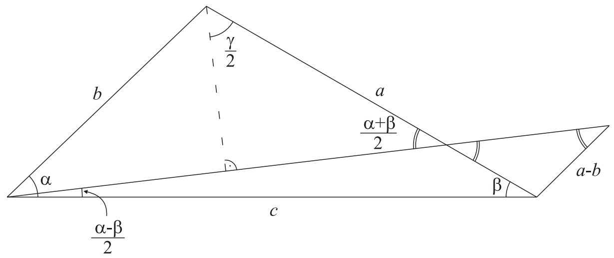

Figure 1: *Mollweide equation - Proof without Words. According DeKleine (1988).*

One of the more puzzling aspects is why these equations should have become known as the Mollweide equations since in the 1808 paper in which they appear Mollweide refers the book by Antonio Cagnoli (1743-1816) *Traité de Trigonométrie Rectiligne et Sphérique, Contenant des Méthodes et des Formules Nouvelles, avec des Applications à la Plupart des Problêmes de l'astronomie* (1786) which contains the formulas. However, the formulas go back to Isaac Newton, or even earlier, but there is no doubt that Mollweide's discovery was made independently of this earlier work (URL1).

2 Mollweide Map Projection Equations

Pseudocylindrical map projections have in common straight parallel lines of latitude and curved meridians. Until the 19th century the only pseudocylindrical projection with important properties was the sinusoidal or Sanson-Flamsteed. The sinusoidal has equally spaced parallels of latitude, true scale along parallels, and equivalency or equal-area. As a world map, it has disadvantage of high distortion at latitudes near the poles, especially those farthest from the central meridian (Fig. 2).

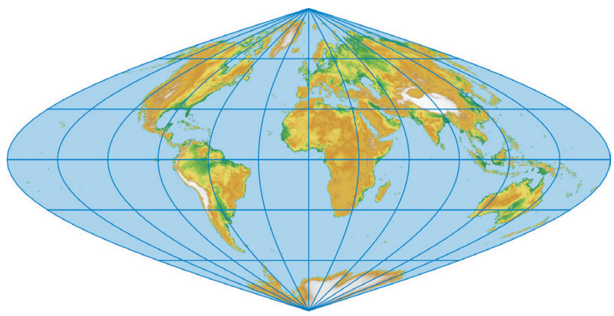

Figure 2: *Sanson or Sanson-Flamsteed or Sinusoidal projection*

In 1805, Mollweide announced an equal-area world map projection that is aesthetically more pleasing than the sinusoidal because the world is placed in an ellipse with axes in a 2:1 ratio and all the meridians are equally spaced semiellipses. The Mollweide projection was the only new pseudocylindrical projection of the nineteenth century to receive much more than academic interest (Fig. 3).

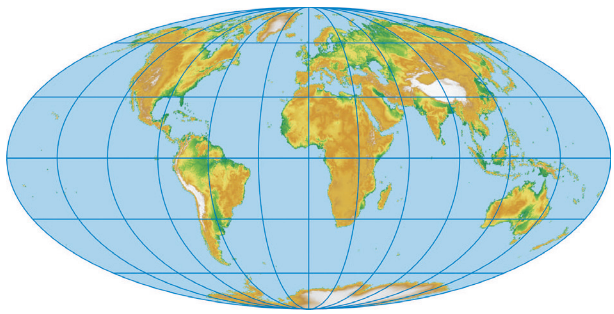

Figure 3: *Mollweide projection*

Mollweide presented his projection in response to a new globular projection of a hemisphere, described by Georg Gottlieb Schmidt (1768-1837) in 1803 and having the same arrangement of equidistant semiellipses for meridians. But Schmidt's curved parallels do not provide the equal-area property that Mollweide obtained (Snyder, 1993).

O'Connor and Robertson (URL1) stated that Mollweide produced the map projection to correct the distortions in the Mercator projection, first used by Gerardus Mercator in 1569. While the Mercator projection is well adapted for sea charts, its very great exaggeration of land areas in high latitudes makes it unsuitable for most other purposes. In the Mercator projection the angles of intersection between the parallels and meridians, and the general configuration of the land, are preserved but as a consequence areas and distances are increasingly exaggerated as one moves away from the equator. To correct these defects, Mollweide drew his elliptical projection; but in preserving the correct relation between the areas he was compelled to sacrifice configuration and angular measurement. The Mollweide projection lay relatively dormant until J. Babinet reintroduced it in 1857 under the name homalographic. The projection has been also called the Babinet, homalographic, homolographic and elliptical projection. It is discussed in many articles, see for example Boggs (1929), Close (1929), Feeman (2000), Philbrick (1953), Reeves (1904) and Snyder (1977) and books or textbooks by Fiala (1957), Graur (1956), Kavrajskij (1960), Kuntz (1990), Maling (1980), Snyder (1987, 1993), Solov'ev (1946) and Wagner (1949). The well known equations of the Mollweide projections read as follows:

$$x = \sqrt{2}R\sin\beta \tag{1}$$

$$y = \frac{2\sqrt{2}}{\pi}R\lambda\cos\beta \tag{2}$$

$$2\beta + \sin 2\beta = \pi\sin\varphi. \tag{3}$$

In these formulas x and y are rectangular coordinates in the plane of projection, φ and λ are geographic coordinates of the points on the sphere and R is the radius of the sphere to be mapped. The angle β is an auxiliary angle that is connected with the latitude φ by the relation (3). For given latitude φ, the equation (3) is a transcendental equation in β. In the past, it was solving by using tables and interpolation method. In our days, it is usually solved by using some iterative numerical method, like bisection or Newton-Raphson method.

2.1 First approach

A half of the sphere with the radius R should be mapped onto the disk with the radius ρ (adopted from Borčić, 1955). If we request that the area of the hemisphere is equal to the area of the disk, than there is the following relation:

$$2R^2\pi = \rho^2\pi \tag{4}$$

from where we have

$$\rho = \sqrt{2}R. \tag{5}$$

Let the circle having the radius ρ be the image of the meridians with the longitudes $\lambda = \pm\frac{\pi}{2}$. From Fig. 4 we see that the rectangular coordinates x_0 and y_0 of any point T_0 belonging to this circle can be written like this:

$$x_0 = \rho \sin\beta \tag{6}$$

$$y_0 = \rho \cos\beta. \tag{7}$$

Due to the request that the projection should be pseudocylindrical, the abscise $x = x_0$ for any point with the same latitude regardless of the longitude the relation (1) holds.

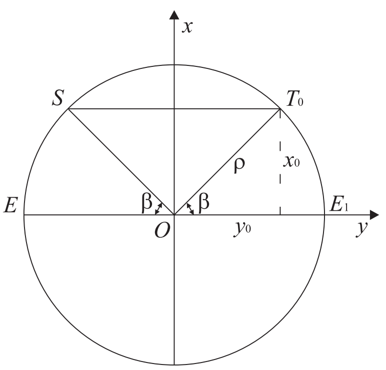

Figure 4: *Derivation of Mollweide projection equations*

On the other hand, the ordinate y will depend on the latitude and longitude. According to the equal-area condition, the following relation exists:

$$y_0 : y = \frac{\pi}{2} : \lambda. \tag{8}$$

By using (8) and (5), the relation (7) goes into (2). In order to finish the derivation, we need to find the relation between the auxiliary angle β, and the latitude φ. According to the equal-area condition, the area SEE_1T_0 should be equal to the area of the spherical segment between the equator and the parallel of latitude φ, which is mapped as the straight-line segment ST_0:

$$\Delta OST_0 + 2OT_0E_1 = R^2\pi\sin\varphi,$$

that is

$$\frac{\rho^2}{2}\sin(\pi - 2\beta) + \frac{2\rho}{2}\beta\rho = R^2\pi\sin\varphi \tag{9}$$

from where we have (3).

2.2 Second approach

Given the earth's radius R, suppose the equatorial aspect of an equal-area projection with the following properties:

- A world map is bounded by an ellipse twice broader than tall
- Parallels map into parallel straight lines with uniform scale
- The central meridian is a part of straight line; all other ones are semielliptical arcs.

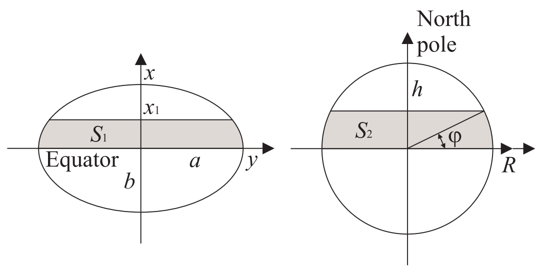

Figure 5: *Second approach to derivation of Mollweide projection equations*

Suppose an earth-sized map; let us define two regions, S_1 on the map and S_2 on the earth, both bounded by the equator and a parallel (URL2). The equal-area property can be used to calculate x for given φ. Given x and λ, y can be calculated immediately from the ellipse equation, since horizontal scale is constant.
Equation of ellipse centred in origin, with major axis on y-axis is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ or}$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right).$$

For $0 \leq x \leq a$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}.$$

Area between y-axis and parallel mapped into $x = x_1$ is

$$S_1 = 2\int_0^{x_1} y dx = 2\frac{b}{a}\int_0^{x_1} \sqrt{a^2 - x^2} dx$$

Let $x = a\sin\beta, 0 \leq x \leq a, 0 \leq \beta \leq \frac{\pi}{2}, dx = a\cos\beta d\beta$, then

$$\int \sqrt{a^2 - x^2} dx = \int \sqrt{a^2(1-\sin^2\beta)}\, a\cos\beta d\beta =$$

$$a^2 \int \cos^2\beta d\beta.$$

Since $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

$$a^2 \int \cos^2\beta d\beta = a^2 \int \frac{1+\cos 2\beta}{2} d\beta =$$

$$= \frac{a^2}{2}\left(\int d\beta + \int \cos 2\beta d\beta\right) = \frac{a^2}{2}\left(\beta + \frac{\sin 2\beta}{2}\right) + C$$

$$S_1 = 2\frac{b}{a}\left(\frac{a^2}{2}\left(\beta + \frac{\sin 2\beta}{2}\right) + C\right)_0^{\beta} =$$

$$= \frac{ab}{2}(2\beta + \sin 2\beta) = 2R^2(2\beta + \sin 2\beta)$$

for some $0 \leq \beta \leq \frac{\pi}{2}$, corresponding to $x_1 = a\sin\beta$ and because of $ab\pi = 4R^2\pi$.

On a sphere, the area between the equator and parallel φ is

$$S_2 = 2\pi Rh = 2\pi R^2 \sin\varphi$$

$$S_1 = S_2 \;\Rightarrow\; 2R^2(2\beta + \sin 2\beta) = 2\pi R^2 \sin\varphi, \text{ i.e. (3).}$$

The auxiliary angle β must be found by interpolation or successive approximation. Finally, since horizontal scale is uniform, and $ab\pi = 4R^2\pi$, $b = 2a$ and $a = \sqrt{2}R$ we have (1). Due to the relation

$$y : \lambda = \frac{b}{a}\sqrt{a^2 - x^2} : \pi$$

$$y = \frac{2\lambda}{\pi}\sqrt{2R^2 - x^2} = 2\sqrt{2R^2 - 2R^2\sin^2\beta}\,\frac{\lambda}{\pi}, \text{ i.e. (2) holds.}$$

2.3 Third approach

From the theory of map projections we know that general equations of pseudocylindrical projections have the form:

$$x = x(\varphi) \tag{10}$$

$$y = y(\varphi, \lambda) \tag{11}$$

Furthermore, for equal-area pseudocylindrical projection holds (Borčić, 1955)

$$y = \frac{R^2\cos\varphi}{\frac{dx}{d\varphi}}\lambda \tag{12}$$

Let us suppose that a half of the sphere has to be mapped onto a disc with the boundary

$$x^2 + y^2 = \rho^2.$$

In order to have an equal-area mapping of the half of the sphere with the radius R onto a disc with the radius ρ we should have

$$2R^2\pi = \rho^2\pi$$

from where

$$\rho^2 = 2R^2.$$

That implies

$$x^2 + y^2 = 2R^2.$$

Taking into account (12) for $\lambda = \pm\frac{\pi}{2}$

$$y = \pm\frac{R^2\pi}{2}\frac{\cos\varphi}{\frac{dx}{d\varphi}}$$

$$x^2 + \frac{R^4\pi^2}{4}\frac{\cos^2\varphi}{\left(\frac{dx}{d\varphi}\right)^2} = 2R^2.$$

That is a differential equation that could be solved by the method of separation of variables:

$$2\sqrt{2R^2 - x^2}dx = R^2\pi\cos\varphi d\varphi$$

where the sign + has been chosen. After integration we can get

$$2\int \sqrt{2R^2 - x^2}dx = R^2\pi\sin\varphi + C$$

By the appropriate substitution in the integral on the left side, or just looking to any mathematical manual we can get the following:

$$2\cdot\frac{1}{2}\left(x\sqrt{2R^2 - x^2} + 2R^2\arcsin\frac{x}{R\sqrt{2}}\right) = R^2\pi\sin\varphi + C$$

For $\varphi = 0, x = 0$ and $C = 0$.
Therefore we have

$$x\sqrt{2R^2 - x^2} + 2R^2\arcsin\frac{x}{R\sqrt{2}} = R^2\pi\sin\varphi. \tag{13}$$

By substitution (1), (13) goes to (3), while (12) can be written as

$$y = \frac{2\lambda}{\pi}\sqrt{2R^2 - x^2}, \text{ which is equivalent to (2).}$$

Remark 1
Although the applied condition was that a half of the sphere has to be mapped onto a disc, the final projection equations hold for the whole sphere and give its image situated into an ellipse.

Remark 2
In references, the Mollweide projection is always defined by equations (1)-(3), which means by using an auxiliary angle or parameter. My equation (13) shows that there is no need to use any auxiliary parameter. There exists the direct relation between the x-coordinate and the latitude φ.

Remark 3
The method applied in this chapter can be applied in derivation of other pseudocylindrical equal-area projections, as are e.g. Sanson projection, Collignon projection or even cylindrical equal-area projection.

3 Generalization of Mollweide Projection

Let us consider the shape of the Mollweide projection of the whole sphere. From the equations (1) and (2), by elimination of β it is easy to obtain the equation of a meridian in the projection

$$\left(\frac{x}{\sqrt{2}R}\right)^2 + \left(\frac{\pi y}{2\sqrt{2}R\lambda}\right)^2 = 1. \tag{14}$$

It is obvious that for a given λ (14) is the equation of an ellipse. It follows that the semiaxis a is constant, while b depends on the longitude λ. If we take $\lambda = \pi$, than $b = 2\sqrt{2}R$, and

$$a : b = 1 : 2 \tag{15}$$

and that is the ratio of semiaxes in the Mollweide projection. The question arises: is it possible to find out a pseudocylindrical equal-area projection that will give the whole word in an arbitrary ellipse satisfying any given ratio $a : b$ or $b : a$? The answer is yes, and we are going to proof it. Let us denote $\mu = b : a$. First of all, the area of an ellipse with the semiaxes a and $b = \mu a$ should be equal to the area of the whole sphere:

$$ab\pi = \mu a^2\pi = 4R^2\pi.$$

This is equivalent with

$$b = \frac{4R^2}{a}, \mu = \frac{4R^2}{a^2} \text{ or } a = \tfrac{2R}{\sqrt{\mu}}. \tag{16}$$

Now, the equation of the ellipse with the centre in the origin and with the semiaxes a and b reads

$$\frac{x^2}{a^2} + \frac{y^2}{\mu^2 a^2} = 1, \text{ or}$$

$$y^2 = \mu^2(a^2 - x^2). \tag{17}$$

Furthermore, the projection should be cylindrical and equal-area, which is generally expressed by (12). If we substitute (12) into (17), taking into account that $\lambda = \pi$, after some minor transformation we can get the following differential equation with separated variables

$$R^2\pi\cos\varphi d\varphi = \mu\sqrt{a^2 - x^2}dx. \tag{18}$$

Integral of the left side of the equation is elementary, while for that on the right side we need a substitution

$$x = a\sin\beta. \tag{19}$$

This leads to the equation

$$\pi\cos\varphi d\varphi = 4\cos^2\beta d\beta.$$

The application of the trigonometric identity

$$\cos^2\beta = \frac{1+\cos 2\beta}{2}$$

gives us the following differential equation that is ready for integration:

$$\pi\cos\varphi d\varphi = 2(1+\cos 2\beta)d\beta.$$

After integration, we obtain

$$\pi\sin\varphi = 2\beta + \sin 2\beta + C, \tag{20}$$

where C is an integration constant. By using the natural conditions $\varphi = 0, x = 0$ and $\beta = 0$ we obtain $C = 0$. In that way, the final form of (5.8) is again the known relation (3). From (18) and (19) we have

$$\frac{dx}{d\varphi} = \frac{a^2\pi\cos\varphi}{4\sqrt{a^2 - x^2}} = \frac{a\pi\cos\varphi}{4\cos\beta} = \frac{R}{\sqrt{\mu}}\frac{\pi\cos\varphi}{2\cos\beta}$$

and taking into account (12)

$$y = \frac{4R^2}{a\pi}\lambda\cos\beta = \mu a\frac{\lambda}{\pi}\cos\beta = 2R\sqrt{\mu}\frac{\lambda}{\pi}\cos\beta,$$

while

$$x = a\sin\beta = \frac{2R}{\sqrt{\mu}}\sin\beta.$$

Let us summarize:

$$x = \frac{2R}{\sqrt{\mu}} \sin\beta.$$

$$y = 2R\sqrt{\mu}\frac{\lambda}{\pi}\cos\beta.$$

$$2\beta + \sin 2\beta = \pi \sin\varphi.$$

These are equations defining the generalized Mollweide projection onto an ellipse of any given ratio $\mu = b : a$ of its semiaxes.

Example 1.
Let us take $\mu = 1$, that is $a = b$, which means that we have a bounding circle. According to (16) $a = b = 2R$.

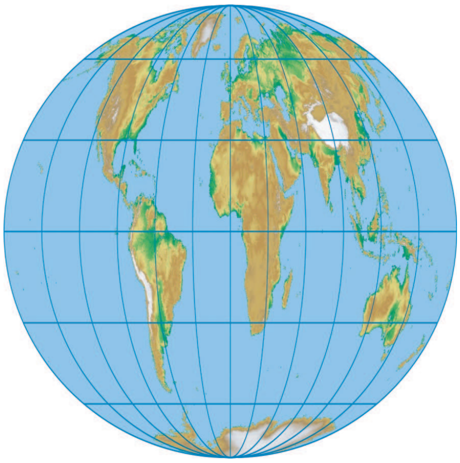

Figure 6: *Generalized Mollweide projection onto a disc*

Example 2.
Let us take $\mu = 2$, that is $b = 2a$. According to (16) $a = \sqrt{2}R, b = 2\sqrt{2}R$, and we are able to recognize the classic Mollweide projection (Fig. 3).

Example 3.
Let us define the ratio μ, by the condition that the linear scale along the equator equals 1. From the theory of map projections it is known that the linear scale along parallels is given by

$$n = \frac{\sqrt{G}}{R\cos\varphi},$$

where

$$G = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2.$$

In our case $x = x(\varphi)$, which means that

$$\frac{\partial x}{\partial \lambda} = 0.$$

The condition

$$n = 1 \text{ for } \varphi = 0$$

goes to

$$\frac{\partial y}{\partial \lambda} = R\cos\varphi = R.$$

Now,

$$\frac{\partial y}{\partial \lambda} = \frac{2R\sqrt{\mu}}{\pi}\cos\beta = R.$$

and from there and $\beta = 0$ due to $\varphi = 0$ we have

$$\sqrt{\mu} = \frac{\pi}{2}, \text{ or } \mu = \tfrac{\pi^2}{4}.$$

Finally, $a = \frac{4R}{\pi}$, $b = R\pi$ and

$$x = \frac{4}{\pi}R\sin\beta$$

$$y = R\lambda\cos\beta$$

$$2\beta + \sin 2\beta = \pi\sin\varphi.$$

It is easy to see that the linear scale in the direction of meridian is also 1 throughout the equator in this version of Mollweide projection (Fig. 7). See also Bromley (1965).

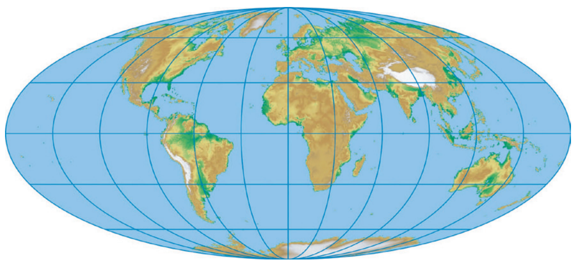

Figure 7: *Generalized Mollweide projection without linear distortions along the equator*

Remark 4
The same approach can be applied to find a generalized Mollweide projection satisfaying the condition $n = 1$ for $\varphi = \varphi_0$, where $0 \le \varphi_0 \le \frac{\pi}{2}$.

4 Inverse Equations of Mollweide Projection

The inverse equations of any map projections read as follows:

$$\varphi = \varphi(x,y)$$

$$\lambda = \lambda(x,y).$$

The computation of φ and λ from given x and y in Mollweide projection is straightforward. In fact, for the given x from (1) we can get the auxiliary angle β

$$\sin\beta = \frac{x}{\sqrt{2}R}$$

Then, from (3) we have

$$\sin\varphi = \frac{1}{\pi}(2\beta + \sin 2\beta)$$

and from (2)

$$\lambda = \frac{\pi y}{2\sqrt{2}R\cos\beta}.$$

5 Distribution of Distortions in Mollweide Projection

For the Mollweide projection given by equations (1)–(3) it can be derived in the straightforward manner:

$$\tan\varepsilon = \frac{2\tan\beta}{\pi}\lambda$$

$$m = \frac{\pi\cos\varphi}{2\sqrt{2}\cos\beta\cos\varepsilon}$$

$$n = \frac{2\sqrt{2}\cos\beta}{\pi\cos\varphi}$$

$$2\tan\frac{\omega}{2} = \sqrt{m^2 + n^2 - 2},$$

where

ε is defined by $\varepsilon = \theta - \frac{\pi}{2}$, and θ is the angle between a meridian and a parallel in the plane of projection
m is a linear scale along meridian
n is a linear scale along parallel
ω is a maximal angular distortion at a point.

The scale of the area $p = 1$ by definition.

The distribution of distortion of Mollweide projection has been investigated and represented in tabular and/or graphical form by several authors (Behrmann, 1909, Solov'ev, 1946, Graur, 1956, Fiala, 1957, Maling 1980).

The linear scale along parallels depends on latitude only. The linear scale along meridians depends both on latitude and longitude. The only standard parallels are 40°44'12"N and S. The only two points with no distortion are the intersections of the central meridian and standard parallels.

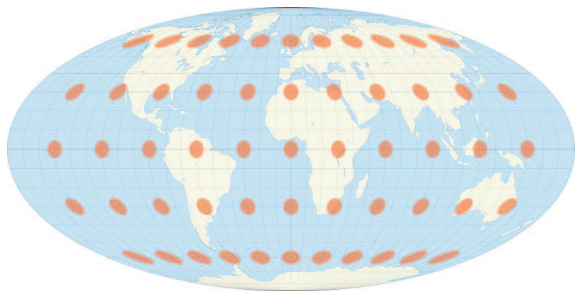

Figure 8: *The Mollweide projection with Tissot's indicatrices of deformation (URL5)*

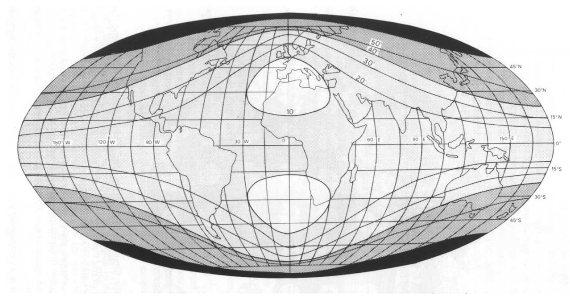

Figure 9: *Mollweide projection for the whole word, showing isograms for maximum angular deformation at* 10°, 20°, 30°, 40° *and* 50°. *Parts of the world map where* $\omega > 80°$ *are shown in black (Maling, 1980; Canters and Crols, 2011).*

6 Some Applications of Mollweide Projection

For those who would like to research the Mollweide projection in more detail, I would recommend the following web-sites: URL2, URL3 and URL6. Although, due it carefully, due to some incorrect statements occurring on the Internet.

Mollweide's projection has been extremely influential. Besides the developments by Goode (URL7), derived works include the interrupted Sinu-Mollweide projection by A. K. Philbrick (1953), other aspect maps like Bartholomew's *Atlantis*, and simple rescaling by reciprocal factors which preserve its features - e.g., making the equator a standard parallel free of distortion (Bromley, 1965), or making the whole map circular instead of elliptical as indicating in the Chapter 3.

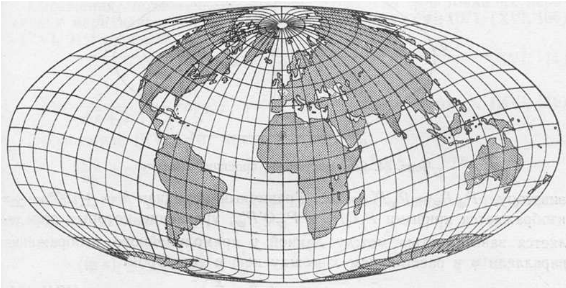

Figure 10: *Oblique aspect of the Mollweide projection (Solov'ev, 1946, Kavrajskij, 1960*

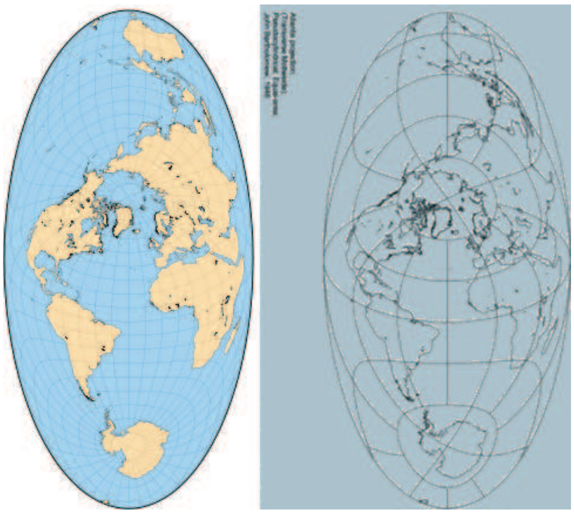

Figure 11: *The Atlantis Map (Bartholomew, 1948), Transversal aspect of the Mollweide projection (URL3)*

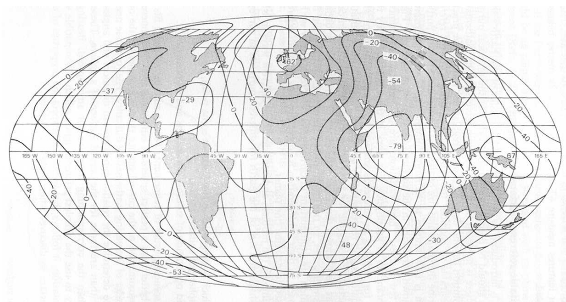

Figure 12: *Inferred contours of the geoid (in metres) for the whole word, based upon Kuala's analysis of variations in gravity potential with both latitude and longitude (Maling 1980)*

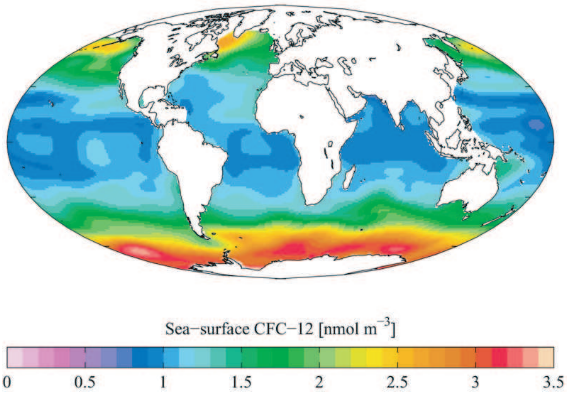

Figure 13: *Sea-surface freon levels measured by the Global Ocean Data Analysis Project. Projected using the Mollweide projection (URL5).*

Figure 14: *The Map Room - A weblog about maps (URL8)*

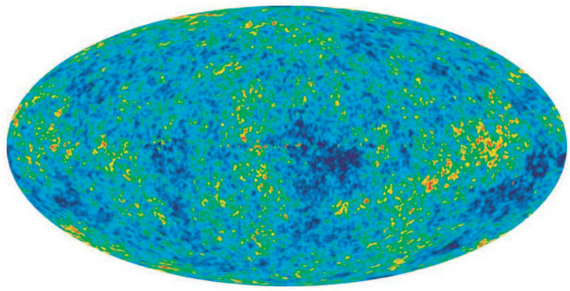

Figure 15: *Full-sky image of Cosmic Microwave Background as seen by the Wilkinson Microwave Anisotropy Probe (URL5).*

Remark 5

The Mollweide and Hammer projections are occasionally confused, since they are both equal-area and share the elliptical boundary; however, the latter design has curved parallels and is not pseudocylindrical (Fig. 16).

The logo of the International Cartographic Associtaion (ICA) has the world in Mollweide projection in its central part (Fig. 17). The mission of the ICA is to promote the discipline and profession of cartography in an international context.

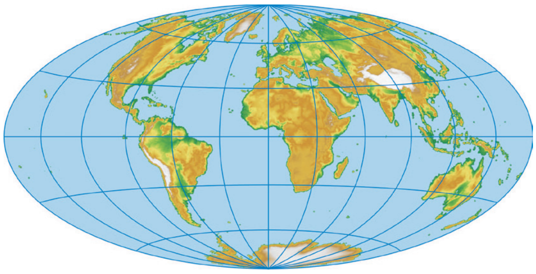

Figure 16: *Hammer projection (URL 9)*

Figure 17: *ICA logo (URL4)*

7 Conclusions

German mathematician and astronomer Karl Brandan Mollweide (1774-1825) is known for trigonometric formulae and map projections named after him. It is possible to derive his projection equations in different ways. One can choose the classic approach without using calculus, another using integrals or the third one, which consists of establishing and solving a differential equation.

Furthermore, it is possible to generalize the Mollweide projection in order to provide pseudocylindrical equal-area projections which represent the entire Earth in an ellipse with any prescribed ratio of its semiaxes. The original Mollweide projection has the ratio of 2:1. Inverse equations of Mollweide projection also exist.

The paper also provides the formulae and illustrations of the distortion distribution in the Mollweide projection. Considering several applications of Mollweide projections represented in the paper, it is obvious that even though the map projection is more than 200 years old, it still has numerous applications. For example, the International Cartographic Association has used it in its logo since it was founded 50 years ago.

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